STATISTICS

Form 5

Vol 11

Part 4B - Box and Whisker Diagram

1.	D

1. The required mean height

$= \frac{162+(166)(2)+(170)(2)+(176)(2)+190}{8}$

= 172 cm

2. (a) Since the range of the distribution is 28 cm, we have

$159 - (130 + a) = 28$

$a = 1$

Since the mean of the distribution is 145 cm, we have

$\frac{131+132+134+141+(140+b)+143+143+144+144+145+147+151+151+156+158+159}{16} = 145$

$b = 1$

The required inter-quartile range

$= \frac{151+151}{2} - \frac{141+141}{2}$

= 10 cm

(b) (i) The inter-quartile range of the distribution of heights of the students in group *B*

= 168 – 157

= 11

The inter-quartile range of the distribution of heights of the students in group *B* is more than the inter-quartile range of the distribution of heights of the students in group *A*.

Thus, the distribution of heights of the students in group *B* is more dispersed than that in group *A*.

(ii) The median (162 cm) of the distribution of heights of the students in group *B* is greater than the maximum (159 cm) of the distribution of heights of the students in group *A*.

Hence, at least half of the students in group *B* are taller than the students in group *A*.

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3. (a) Assume the marks scored by the 10 participants in group A are arranged in ascending order as follows: 58, x_1, 63, x_2, x_3, x_4, x_5, 80, x_6, 96.

When $x_1 = 63$, $x_2 = x_3 = x_4 = 78$, $x_5 = 80$ and $x_6 = 96$, the mean mark is the greatest.

$$m \le \frac{58+63+63+78+78+78+80+80+96+96}{10}$$

$m \le 77$

When $x_1 = 58$, $x_2 = 63$, $x_3 = x_4 = x_5 = 78$ and $x_6 = 80$, the mean mark is the least.

$$m \ge \frac{58+58+63+63+78+78+78+80+80+96}{10}$$

$m \ge 73.2$

Thus, we have $73.2 \le m \le 77$.

(b) (i) Note that the median of the marks of the participants in group B is 78.

So, we have $\frac{(70+b)+79}{2} = 78$.

Hence, we have $b = 7$.

The range of the marks of the participants in group $B = 96 - 58 = 38$.

So, we have $(90 + d) - 60 = 38$.

Hence, we have $d = 8$.

The inter-quartile range of the marks of the participants in group $B = 80 - 63 = 17$.

So, we have $\frac{(80+c)+87}{2} - \frac{62+(70+a)}{2} = 17$.

Hence, we have $a - c = 1$.

Also note that $0 \le a \le 3$ and $1 \le c \le 7$.

Thus, we have $\begin{cases} a = 2 \\ b = 7 \\ c = 1 \\ d = 8 \end{cases}$ or $\begin{cases} a = 3 \\ b = 7 \\ c = 2 \\ d = 8 \end{cases}$.

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(ii) When $a = 2$, $b = 7$, $c = 1$ and $d = 8$,

the mean of the marks of the participants in group B

$$= \frac{60+61+62+72+73+77+79+81+81+87+91+98}{12}$$

$$= \frac{461}{6}$$

Hence the mean of the marks of the participants in group A is $\frac{461}{6}$.

Then, the sum of the marks of the participants in group $A = \left(\frac{461}{6}\right)(10) = \frac{2305}{3}$,

which is impossible.

Thus, we have $a = 3$, $b = 7$, $c = 2$ and $d = 8$.

Hence, the marks of the 12 participants in group B are: 60, 61, 62, 73, 73, 77, 79, 81, 82, 87, 91 and 98.

The mean of the marks of the participants in group A

$$= \frac{60+61+62+73+73+77+79+81+82+87+91+98}{12}$$

$= 77$

So, the marks of the 10 participants in group A are: 58, 63, 63, 78, 78, 78, 80, 80, 96 and 96.

The required probability

$$= 1 - \frac{3+3+3+6+6+6+4+4+2+2}{10 \times 12}$$

$$= \frac{27}{40}$$

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Part 5A – Data Change

1. C	2. D	3. C	4. B	5. C	6. C	7. A
8. B	9. D	10. C	11. C	12. A	13. D	14. D

1. Let μ and σ be mean and the standard deviation of the group of numbers $\{-4d, -2d, 0, 2d, 4d\}$.

   Then, we have $\mu = 0$ and $\sigma = 2\sqrt{2}\,d$.

   Note that the five numbers given can be formed by adding a to each number in $\{-4d, -2d, 0, 2d, 4d\}$.
   The required standard deviation
   $= \sigma$
   $= 2\sqrt{2}\,d$

2. Let μ_1 and σ_1 be the mean and the standard deviation of the group of numbers $\{-4, -3, -2, -1, 1, 2, 3, 4\}$ respectively while μ_2 and σ_2 be the mean and the standard deviation of the group of numbers $\{-8, -6, -4, -2, 2, 4, 6, 8\}$ respectively.
   Note that $\{-8, -6, -4, -2, 2, 4, 6, 8\}$ can be formed by multiplying each number in $\{-4, -3, -2, -1, 1, 2, 3, 4\}$ by 2.
   So, we have $\mu_1 = \mu_2 = 0$ and $\sigma_2 = 2\sigma_1$.

   Further note that A can be formed by adding a to each number in $\{-4, -3, -2, -1, 1, 2, 3, 4\}$ while B can be formed by adding a to each number in $\{-8, -6, -4, -2, 2, 4, 6, 8\}$.

   So, we have $\begin{cases} m_1 = \mu_1 + a = a \\ s_1 = \sigma_1 \end{cases}$ and $\begin{cases} m_2 = \mu_2 + a = a \\ s_2 = \sigma_2 \end{cases}$.

   Thus, we have $m_1 = m_2$ and $2s_1 = s_2$.

3. Let μ_1, m_1 and v_1 be the mean, the median and the inter-quartile range of the group of numbers $\{-8, -5, -2, 0, 7, 8\}$ respectively while μ_2, m_2 and v_2 be the mean, the median and the inter-quartile range of the group of numbers $\{-7, -4, -3, 1, 4, 9\}$ respectively.

   Then, we have $\begin{cases} \mu_1 = 0 \\ m_1 = -1 \\ v_1 = 12 \end{cases}$ and $\begin{cases} \mu_2 = 0 \\ m_2 = -1 \\ v_2 = 8 \end{cases}$.

   Hence, we have $\mu_1 = \mu_2$, $m_1 = m_2$ and $v_1 \neq v_2$.

   Note that $\{a - 8, a - 5, a - 2, a, a + 7, a + 8\}$ can be formed by adding a to each number in $\{-8, -5, -2, 0, 7, 8\}$ while $\{a - 7, a - 4, a - 3, a + 1, a + 4, a + 9\}$ can be formed by adding a to each number in $\{-7, -4, -3, 1, 4, 9\}$.
   So, the two groups of numbers have the same mean and the same median, but different inter-quartile range.
   Thus, I and II only.

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4. The mean of the new set of numbers = $10m + 7$
The range of the new set of numbers = $10r$
The variance of the new set of numbers = $(10)^2 v = 100v$

5. The median of the new set of numbers = $5(p + 5) = 5p + 25$
The inter-quartile range of the new set of numbers = $5q$.

6. The mode of the new set of numbers = $2(37 + 9) = 92$
The variance of the new set of numbers = $(2)^2(8) = 32$
The inter-quartile range of the new set of numbers = $(2)(16) = 32$

7. Note that $\{x_1 + 3, x_2 + 3, x_3 + 3, x_4 + 3, x_5 + 3\}$ can be formed by adding 3 to each number in $\{x_1, x_2, x_3, x_4, x_5\}$.
Thus, the required standard deviation is 10.

8. Note that $\{3x_1 - 1, 3x_2 - 1, 3x_3 - 1, 3x_4 - 1, 3x_5 - 1\}$ can be formed by multiplying 3 to each number in $\{x_1, x_2, x_3, x_4, x_5\}$ and followed by adding –1 to each resulting number.
The required standard deviation
$= (3)(10)$
$=30$

9. Note that $\{7 - 3x_1, 7 - 3x_2, 7 - 3x_3, 7 - 3x_4, 7 - 3x_5, 7 - 3x_6\}$ can be formed by multiplying –3 to each number in $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ and followed by adding 7 to each resulting number.
So, we have $m_2 = 7 - 3m_1$, $r_2 = 3r_1$ and $s_2 = 3s_1$.
Hence, we have $r_1 < r_2$ and $s_1 < s_2$.
However, the statement "$m_1 > m_2$" cannot be determined from the information provided.

Thus, II and III only.

10. Note that $\{a, b, c, d\}$ can be formed by adding –1 to each number in $\{1 - 3a, 1 - 3b, 1 - 3c, 1 - 3d\}$ and followed by multiplying $-\frac{1}{3}$ to each resulting number.
The required variance
$= \left(\frac{1}{3}\right)^2 (27)$
$= 3$

11. The required variance
$= (2)^2(16)$
$= 64$

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12. Denote the arithmetic sequence by $a_n = a + (n - 1)d$, where $1 \leq n \leq 52$, a represents the first term of the sequence and d represents the common difference of the sequence.
Note that $\{a_{27}, a_{28}, a_{29}, \dots, a_{52}\}$ can be formed by adding $26d$ to each term in $\{a_1, a_2, a_3, \dots, a_{26}\}$.
So, the variance of the last 26 terms of the sequence is 25.

The required standard deviation

$= \sqrt{25}$

$= 5$

13. Note that $\{3a - 12, 12b - 21, 6c - 36, 27\}$ can be formed by adding -7 to each number in $\{a + 3, 4b, 2c - 5, 16\}$ and followed by multiplying 3 to each resulting number.
The required variance
$= (3)^2(11)$
$= 99$

14. Note that $z = -1.6$.
$x(1 + 25\%) + 7 = 52$
$x = 36$

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