CONGRUENT AND SIMILAR TRIANGLES

Form 1 Regular Course
Vol 9

Part 5 – Using similar triangles

1. $q = \angle DFE = \angle CBA = 45°$ (corr. ∠s, ~Δs)
   $p = 180° - 65° - q = 70°$ (∠ sum of Δ)
   $r = \angle BAC = \angle FED = q = 70°$ (corr. ∠s, ~Δs)
   $s = \angle ACB = \angle EDF = 65°$ (corr. ∠s, ~Δs)

2. $\frac{10}{5} = \frac{a}{13} = \frac{24}{b}$ (corr. sides, ~Δs)
   $a = 26$
   $b = 12$

3. $\angle CBD = \angle CEA = 48°$ (corr. ∠s, ~Δs)
   $\angle BDC = 180° - 34° - 48° = 98°$ (∠ sum of Δ)

4. $\frac{XY}{8} = \frac{AX}{AB} = \frac{1}{2}$ (corr. sides, ~Δs)
   $XY = 4$ cm

5. $\frac{x+2+5}{4} = \frac{AD}{CD} = \frac{BD}{ED} = \frac{11+4}{5}$ (corr. sides, ~Δs)
   $x = 5$
   $\frac{1-y}{2} = \frac{AB}{CE} = \frac{BD}{ED} = \frac{11+4}{5}$ (corr. sides, ~Δs)
   $y = -5$

6. $\dfrac{CD}{4}=\dfrac{CD}{CA}=\dfrac{AC}{BC}=\dfrac{4}{16}$ (corr. sides, ~Δs)

$CD = 1$ cm

Area of $\Delta ABD=\dfrac{1}{2}(16+1)(4)=34\text{ cm}^2$

7. $\angle AOB = \angle COD = \angle EOF = 90° \div 3 = 30°$ (corr. ∠s, ~Δs)

$\angle POD = 30° + 30° = 60°$

$\angle ODC = \angle OFE = 62°$ (corr. ∠s, ~Δs)

$\angle OPD = 180° - 62° - 60° = 58°$ (∠ sum of Δ*DOP*)

8. $\dfrac{5.5}{8.8}=\dfrac{AC}{AQ}=\dfrac{BC}{PQ}=\dfrac{2CX}{2.2+5.8}$ (corr. sides, ~Δs)

$CX = 2.5$

Part 6 – Prove first

1. (a) $\angle PQR = \angle NQM$ (common)

   $\angle QPR = \angle QNM$ (given)

   Thus, $\Delta PQR \sim \Delta NQM$ (AA)

   (b) $\frac{9+15}{12} = \frac{x}{6} = \frac{12+y}{9}$ (corr. sides, ~Δs)

   $x = 12$

   $y = 6$

2. (a) $\angle ABC = \angle ADE$ (given)

   $\angle ACB = \angle AED$ (given)

   Thus, $\Delta ABC \sim \Delta ADE$ (AA).

   (b) $\frac{3+CE}{3} = \frac{2+4}{2}$ (corr. ∠s, ~Δs)

   $CE = 6$ cm

3. (a) $\angle QPR = \angle NPM$ (common)

   $\angle PQR = \angle PNM$ (given)

   Thus, $\Delta PQR \sim \Delta PNM$ (AA).

   (b) $\frac{2+n}{3} = \frac{4}{2} = \frac{5}{m}$ (corr. sides, ~Δs)

   $m = 2.5$

   $n = 4$

4. (a) $\angle ABC = \angle ACD$ (given)

   $\angle BAC = \angle CAD$ (common)

   $\Delta ABC \sim \Delta ACD$ (AA)

   (b) $\frac{BD+8}{10} = \frac{10}{8}$ (corr. sides, ~Δs)

   $BD = 4.5$ cm

5. (a) $\angle BAC = \angle DAB$ (common)

$\frac{AB}{AD} = \frac{21}{9} = \frac{7}{3}$

$\frac{AC}{AB} = \frac{49}{21} = \frac{7}{3}$

Thus, $\Delta ABC \sim \Delta ADB$ (ratio of 2 sides, inc. ∠)

(b) $\angle ADB = 180° - 60° = 120°$ (adj. ∠s on st. line)

$\angle ABC = \angle ADB = 120°$ (corr. ∠s, ~Δs)

(c) $\frac{BD}{35} = \frac{9}{21}$ (corr. sides, ~Δs)

$BD = 15$

6. (a) $\angle DAE = \angle EBF = 108°$ (given)

$\frac{AD}{AE} = \frac{18}{18 \div 2} = 2$

$BF + CF = BF + 3BF = BC = 18$

$BF = 4.5$ cm

$\frac{BE}{BF} = \frac{18 \div 2}{4.5} = 2$

Thus, $\Delta ADE \sim \Delta BEF$ (ratio of 2 sides, inc. ∠).

(b) (i) Let $\angle ADE = \angle BEF = a$ (corr. ∠s, ~Δs) and $\angle AED = \angle BFE = b$ (corr. ∠s, ~Δs)

$a + b + 108° = 180°$ (∠ sum of Δ)

$a + b = 72°$

$a + b + \angle DEF = 180°$ (adj. ∠s on st. line)

$\angle DEF = 180° - (a + b) = 108°$

(ii) $\frac{DE}{EF} = \frac{AD}{BE} = \frac{18}{18 \div 2} = 2$ (corr. sides, ~Δs)

$\frac{AD}{AE} = \frac{18}{18 \div 2} = 2$

$\angle DEF = \angle DAE = 108°$ (proven)

Thus, $\Delta ADE \sim \Delta EDF$ (ratio of 2 sides, inc. ∠).

(c) $\frac{DE}{EF} = 2$ (proven)

$DE = 2EF = 2x$ cm.

$\frac{DF}{DE} = \frac{DE}{DA}$ (corr. sides, ~Δs)

$DF = \frac{(DE)(DE)}{DA} = \frac{4x^2}{18} = \frac{2x^2}{9}$ cm

7 (a) (i) $\angle EAF = \angle FBG = 90°$ (given)

Let $\angle AFE = x$.

$\angle BFG = 180° - 90° - x = 90° - x$ (adj. ∠s on st. line)

$\angle BGF = 180° - 90° - \angle BFG = 90° - (90° - x) = x$ (∠ sum of Δ)

Thus, $\Delta AEF \sim \Delta BFG$ (AA).

$\angle EAF = \angle HDE = 90°$ (given)

$\angle AEF = \angle BFG = 90° - x$ (corr. ∠s, ~Δs)

$\angle DEH = 180° - 90° - \angle AEF = 90° - (90° - x) = x$ (adj. ∠s on st. line)

Thus, $\Delta AEF \sim \Delta DHE$ (AA).

(ii) $\frac{AE}{AF} = \frac{BF}{BG}$ (corr. sides, ~Δs)

$$\frac{x+1}{3} = \frac{4x}{9}$$

$$3x + 3 = 4x$$

$$x = 3$$

$\frac{AE}{AF} = \frac{DH}{DE}$ (corr. sides, ~Δs)

$$\frac{3+1}{3} = \frac{8}{y}$$

$$y = 6$$

(b) $AD = x + 1 + y = 10$

$AB = 3 + 4x = 15$

$CG = 10 - 9 = 1$

$CH = 15 - 8 = 7$

Area of $EFGH$

$$= (10)(15) - \frac{9(4\times 3)}{2} - \frac{3(3+1)}{2} - \frac{8(6)}{2} - \frac{1(7)}{2}$$

$$= 62.5$$