CONGRUENT AND SIMILAR TRIANGLES

Form 1 Regular Course
Vol 9

Part 3B – Prove first (B)

1. (a) $AB = DC$ (given)
   $AC = DB$ (given)
   $BC = CB$ (common)
   Thus, $\Delta ABC \cong \Delta DCB$ (SSS).
   (b) $AB = DC$ (given)
   $\angle BAE = \angle CDE$ (corr. $\angle$s, $\cong\Delta$s)
   $\angle AEB = \angle DEC$ (vert. opp. $\angle$s)
   Thus, $\Delta ABE \cong \Delta DCE$ (AAS).

2. $\angle BAC = \angle BDC = 90°$ (given)
   $AC = DC$ (given)
   $BC = BC$ (common)
   $\Delta ABC \cong \Delta\ DBC$ (RHS)
   $\angle ABC = \angle DBC$ (corr. $\angle$s, $\cong\Delta$s)

3. (a) $EA = DA$ (given)
   $\angle EAG = \angle DAB = 90°$ (given)
   $\angle EAB = \angle EAB + 90° = \angle DAG$
   $AB = AG$ (given)
   Thus, $\Delta EAB \cong \Delta DAG$ (SAS)
   (b) $EB = DG$ (corr. sides, $\cong\Delta$s)

4. (a) $BC = CB$ (common)
$\angle BCD = \angle CBE = 13°$ (given)
$CD = BE$ (given)
Thus, $\Delta BCD \cong \Delta CBE$ (SAS).
(b) $\angle CBD = \angle BCE$ (corr. $\angle$s, $\cong\Delta$s)
$108° + 2\angle CBD = 180°$ ($\angle$ sum of Δ)
$\angle CBD = 36°$
$\angle BDF = 180° - 13° - 36° = 131°$ ($\angle$ sum of Δ)

5. (a) $\angle ABC = \angle DAE$ (given)
Since $\angle ABC = \angle CAD$ (given),
$\angle BAC = 2\angle DAE = \angle ADE$
$AC = DE$ (given)
Thus, $\Delta ABC \cong \Delta DAE$ (AAS).
(b) $AE = BC = 15$ cm (corr. sides, $\cong\Delta$s)
$AB = AE + BE = 18$ cm
$AD = AB = 18$ cm (corr. sides, $\cong\Delta$s)

6. (a) $AB = CD$ (given)
Since $AE = CF$ (given),
$AF = AE + EF = CF + EF = CE$
$\angle AFB = \angle CED = 90°$ (given)
Thus, $\Delta ABF \cong \Delta CDE$ (RHS).
(b) (i) $DE = BF$ (corr. sides, $\cong\Delta$s)
$\angle AED = 180° - 90° = 90°$ (adj. $\angle$s on st. line)
$\angle CFB = 180° - 90° = 90°$ (adj. $\angle$s on st. line)
$AE = CF$ (given)
Thus, $\Delta ADE \cong \Delta CBF$ (SAS)
(ii) $\angle CAD = \angle ACB$ (corr. $\angle$s, $\cong\Delta$s)
Thus, AD // BC (alt. $\angle$s eq.)

7. (a) $AB = AD$ (given)

$\angle ABC = \angle DAE$ (given)

$\angle ACB = \angle DEA = 90°$ (given)

Thus, $\Delta ABC \cong \Delta DAE$ (AAS).

(b) (i) $AB = AD$ (given)

$\angle BAF = \angle DAF$ (given)

$AF = AF$ (common)

Thus, $\Delta ABF \cong \Delta ADF$. (SAS)

(ii) Let $\angle ABF = \angle DAF = \angle BAF = x$.

$3x + 90° = 180°$ ($\angle$ sum of Δ)

$x = 30°$

$\angle ADE = \angle BAC = 30° + 30° = 60°$ (corr. $\angle$s, $\cong\Delta$s)

$\angle ADF = \angle ABF = 30°$ (corr. $\angle$s, $\cong\Delta$s)

$\angle EDF = \angle ADE - \angle ADF = 30°$

(iii) Notice that $\Delta EDF \cong \Delta CDF$ (AAS) and $\Delta CAF \cong \Delta CDF$ (AAS).

Area of ΔEDF = Area of ΔCDF = Area of $\Delta CAF = k$.

Area of ΔABF = Area of $\Delta ADF = k + k = 2k$

Area of $ADEFB = 2k + 2k + k = 5k$.

Part 4 – Proving similar triangles

1. $\Delta PQR \sim \Delta YXZ$ (3 sides prop.)

2. $\Delta ABC \sim \Delta PRQ$ (3 sides prop.)

3. $\Delta PQR \sim \Delta ZXY$ (AAA)

4. $\Delta ABC \sim \Delta ZXY$ (AAA)

5. $\Delta ABC \sim \Delta XZY$ (ratio of 2 sides, inc. $\angle$)

6. (a) $\angle QRS = \angle TRP$ (common)
   $\angle QSR = \angle TPR$ (given)
   $\Delta QRS \sim \Delta TRP$ (AA)

   (b) $\frac{GD}{DF} = \frac{6}{12} = \frac{1}{2}$

   $\frac{DF}{FE} = \frac{12}{24} = \frac{1}{2}$

   $\frac{GF}{DE} = \frac{9}{18} = \frac{1}{2}$

   $\Delta DEF \sim \Delta GFD$ (3 sides prop.)

   (c) $\frac{AB}{DB} = \frac{4}{3+5} = \frac{1}{2}$

   $\frac{BC}{BE} = \frac{5}{10} = \frac{1}{2}$

   $\angle ABC = \angle DBE$ (given)
   $\Delta ABC \sim \Delta DBE$ (ratio of 2 sides, inc. $\angle$)