STATISTICS

Form 5

Vol 11

Part 2 - Measure of Dispersion

1.	A	2.	C	3.	C	4.	D	5.	A

1. The range of the seven numbers

$= (x-1)-(x-10)$

$= 9$

The inter-quartile range of the seven numbers

$= (x-3)-(x-8)$

$= 5$

2. The mean of the six numbers

$= \dfrac{10+34+48+56+72+89}{6}$

$= 51.5$

The required standard deviation

$= \sqrt{\dfrac{(10-51.5)^2+(34-51.5)^2+(48-51.5)^2+(56-51.5)^2+(72-51.5)^2+(89-51.5)^2}{6}}$

$= \sqrt{\dfrac{7775}{12}}$

≈ 25.5

3. The class marks of the 6 classes are 7, 9, 11, 13, 15 and 17 respectively.

The mean of the distribution

$= \dfrac{(7)(20)+(9)(40)+(11)(25)+(13)(10)+(15)(25)+(17)(15)}{20+40+25+10+25+15}$

$= \dfrac{307}{27}$

The required standard deviation

$= \sqrt{\dfrac{(20)(7-\frac{307}{27})^2+(40)(9-\frac{307}{27})^2+(25)(11-\frac{307}{27})^2+(10)(13-\frac{307}{27})^2+(25)(15-\frac{307}{27})^2+(15)(17-\frac{307}{27})^2}{135}}$

$= \sqrt{\dfrac{7784}{729}}$

≈ 3.27 kg

4. From the bar chart, we have

Score	10	20	30	40	50
Frequency	5	10	20	15	10

Mode	30
Median	30
Lower quartile	25
Upper quartile	40

Thus, D is the desired answer.

5. The mean of the five numbers

$= \dfrac{2x+3x+5x+6x+9x}{5}$

$= 5x$

The required standard deviation

$= \sqrt{\dfrac{(2x-5x)^2+(3x-5x)^2+(5x-5x)^2+(6x-5x)^2+(9x-5x)^2}{5}}$

$= \sqrt{6}\,x$

6. (a) The required range

$= (10.9 + 0.05) - (8.0 - 0.05)$

$= 3$ cm

(b) The class marks of the six classes are 8.2 cm, 8.7 cm, 9.2 cm, 9.7 cm, 10.2 cm and 10.7 cm.

The mean of the lengths of the colour pencils

$= \dfrac{(8.2)(4)+(8.7)(9)+(9.2)(12)+(9.7)(15)+(10.2)(8)+(10.7)(2)}{4+9+12+15+8+2}$

$= 9.4$ cm

The required standard deviation

$= \sqrt{\dfrac{(4)(8.2-9.4)^2+(9)(8.7-9.4)^2+(12)(9.2-9.4)^2+(15)(9.7-9.4)^2+(8)(10.2-9.4)^2+(2)(10.7-9.4)^2}{50}}$

$= \dfrac{\sqrt{41}}{10}$

$= 0.640$ cm

7. (a) (i) The class marks of the four classes are 14.5, 24.5, 34.5 and 44.5.

The required mean

$$= \frac{(14.5)(1.7)+(24.5)(1.9)+(34.5)(2.1)+(44.5)(0.3)}{1.7+1.9+2.1+0.3}$$

$$= \frac{157}{6}$$

The required standard deviation

$$= \sqrt{\frac{(1.7)(14.5-\frac{157}{6})^2+(1.9)(24.5-\frac{157}{6})^2+(2.1)(34.5-\frac{157}{6})^2+(0.3)(44.5-\frac{157}{6})^2}{6}}$$

$$= \frac{5\sqrt{29}}{3}$$

$$\approx 8.98$$

(ii) The required mean

$$= \frac{(14.5)(2.6)+(24.5)(1.4)+(34.5)(0.6)+(44.5)(0.4)}{2.6+1.4+0.6+0.4}$$

$$= 22.1$$

The required standard deviation

$$= \sqrt{\frac{(2.6)(14.5-22.1)^2+(1.4)(24.5-22.1)^2+(0.6)(34.5-22.1)^2+(0.4)(44.5-22.1)^2}{5}}$$

$$= \frac{4\sqrt{141}}{5}$$

$$\approx 9.50$$

(b) From (a), it is found that the standard deviation of the ages of members of Eason's fan club is greater than that of Andy's fan club.

Thus, Eason's fan club has a larger dispersion in the age of members.

8. B	9. D	10. B

8. Note that $1 \le b \le 5$ and $4 \le c \le 7$.

Since the inter-quartile range of the distribution is at most 20 minutes, we have

$$\frac{(40+c)+47}{2} - \frac{(20+a)+(20+a)}{2} \le 20$$

$2a - c \ge 7$

c	**Possible value(s) of a**
4	6, 7, 8, 9
5	6, 7, 8, 9
6	7, 8, 9
7	7, 8, 9

So, we have $6 \le a \le 9$.

Thus, I and III only.

9. Note that $3 \le b \le 5$.

Since the range of the distribution is at most 44, we have

$95 - (50 + a) \le 44$

$a \ge 1$

Also note that $a \le 3$.

So, we have $1 \le a \le 3$.

Hence, we have

$(3) - (3) \le b - a \le (5) - (1)$.

$0 \le b - a \le 4$

Thus, I, II and III.

10. Let μ, m and v be the mean, the median and the variance of the positive integers respectively.

Note that $m = \frac{5^{\text{th}}\text{ datum} + 6^{\text{th}}\text{ datum}}{2}$.

If $p < 12$ or $q < 12$, then we have

$$m = \frac{11+12}{2}$$

$m = 11.5$

However, this contradicts the fact that $m = 12$.

So, we have $p \geq 12$ and $q \geq 12$.

Since the range of the positive integers is 21, the greatest possible value of q is 24.

In this case, $p = 12$.

$$\mu = \frac{3+4+4+11+12+15+20+24+p+q}{10}$$

$$\mu = \frac{93+p+q}{10}$$

$$\mu \leq \frac{93+12+24}{10}$$

$\mu \leq 12.9$

So, it is not always true that $\mu < m$.

$$v = \frac{(3-\mu)^2 + (4-\mu)^2 + (4-\mu)^2 + (11-\mu)^2 + (12-\mu)^2 + (15-\mu)^2 + (20-\mu)^2 + (p-\mu)^2 + (q-\mu)^2}{10}$$

Note that the value of v is the greatest when $p = 12$ and $q = 24$. In this case, $\mu = 12.9$.

So, we have $v \leq 56.29$.

Hence, the variance of the positive integers cannot be 60.

Thus, I and III only.

11. (a) Note that $a = 1$.

Since the range of the distribution is 23, we have $(160 + b) - 140 = 23$.

Hence, we have $b = 3$.

The required mean

$$= \frac{140+141+142+144+149+150+150+151+151+151+153+157+159+161+163}{15}$$

$= 150.8$ cm

Note that the median of the distribution is the eighth datum.

Thus, the required median is 151 cm.

The required standard deviation

≈ 6.85273668

≈ 6.85 cm

(b) The required probability

$= \frac{10}{15}$

$= \frac{2}{3}$

12. (a) Note that the median of the distribution $= \dfrac{9^{\text{th}} \text{ datum} + 10^{\text{th}} \text{ datum}}{2}$.

The required median

$= \dfrac{18+18}{2}$

$= 18$

The required range

$= 36 - 4$

$= 32$

(b) Since the mean of the distribution is 19.5, we have

$$\frac{4+8+9+12+(10+a)+14+15+15+18+18+21+22+26+(20+b)+29+30+34+36}{18} = 19.5$$

$a + b = 10$

Note that $2 \le a \le 4$ and $6 \le b \le 9$.

Thus, we have $\begin{cases} a = 2 \\ b = 8 \end{cases}$ or $\begin{cases} a = 3 \\ b = 7 \end{cases}$ or $\begin{cases} a = 4 \\ b = 6 \end{cases}$.

Note that $(12 - 19.5)^2 + (28 - 19.5)^2 > (13 - 19.5)^2 + (27 - 19.5)^2 > (14 - 19.5)^2 + (26 - 19.5)^2$.

Thus, the standard deviation of the distribution is the greatest when $a = 2$ and $b = 8$.

Now, there are three cases.

Case 1: $a = 2$ and $b = 8$

The standard deviation of the distribution

≈ 9.056918779

Case 2: $a = 3$ and $b = 7$

The standard deviation of the distribution

≈ 8.964435906

Case 3: $a = 4$ and $b = 6$

The standard deviation of the distribution

≈ 8.883505314

Thus, the standard deviation of the distribution is the greatest when $a = 2$ and $b = 8$.

13. (a) Since the inter-quartile range of the distribution is 13 kg, we have

$$\frac{65+65}{2}-\frac{(50+x)+(50+x)}{2}=13$$

$x=2$

(b) (i) First note that $0 \le y \le 4$ and $5 \le z \le 9$.

Since the range of the distribution exceeds 36 kg, we have

$(70+z)-(40+y)>36$

$z-y>6$

Since the mean of the distribution is 59 kg, we have

$$\frac{(40+y)+44+50+50+52+52+54+56+57+58+59+59+61+62+65+65+68+73+75+(70+z)}{20}=59$$

$y+z=10$

Thus, we have $\begin{cases} y=1 \\ z=9 \end{cases}$.

(ii) Let a and b be the weights of the two students, where $a \le b$.

Note that $\frac{a+b+(59)(20)}{22}=59+1$.

So, we have $a+b=140$.

Since the range is increased by 1 kg, the new range = (79 – 41) + 1 = 39 kg.

There are two situations.

Situation 1: $a=40$.

Since $a+b=140$, we have $b=100$.

Then the new range = 100 – 40 = 60 kg, which is impossible.

Situation 2: $41 \le a \le 80$.

Under this circumstance, we have $b=80$.

Since $a+b=140$, we have $a=60$.

Therefore, the weights of the two students are 60 kg and 80 kg.