STATISTICS

Form 5

Vol 11

Part 1 – Measure of Central Tendency

1. C	2. C	3. C	4. A	5. D	6. B	7. B
8. B	9. B	10. D	11. B	12. A	13. B	14. D
15. B	16. A	17. B	18. B	19. B	20. B	21. C
22. B	23. C	24. C				

1. The class marks of the four classes are 157.5 cm, 162.5 cm, 167.5 cm and 172.5 cm.
   The required mean height
   $= \dfrac{(157.5)(7)+(162.5)(14)+(167.5)(11)+(172.5)(8)}{7+14+11+8}$
   = 165 cm

2. The class marks of the five classes are 10.5 seconds, 12.5 seconds, 14.5 seconds, 16.5 seconds and 18.5 seconds.
   The required mean
   $= \dfrac{(10.5)(4)+(12.5)(13)+(14.5)(12)+(16.5)(9)+(18.5)(2)}{4+13+12+9+2}$
   = 14.1 seconds

3. The required mean
   $= 4 \times \dfrac{48°}{360°} + 5 \times \dfrac{75°}{360°} + 7 \times \dfrac{90°}{360°} + 9 \times \dfrac{(360° - 48° - 75° - 90°)}{360°}$
   = 7

4. Note that the median is the 20^{th} datum.
   From the cumulative frequency polygon, the required median is 16.5 kg.

5. From the table, we have

Age	Mark less than	Frequency	Cumulative frequency
6 – 7	7.5	4	4
8 – 9	9.5	7	11
10 – 11	11.5	9	20
12 – 13	13.5	10	30

Thus, D is the desired answer.

6. Observe that $60° < 180°$ and $60° + 135° > 180°$.
Thus, the median of the distribution is 1.

7. Since the mean of the distribution is 1.5, we have
$$0 \times \frac{60°}{360°} + 1 \times \frac{135°}{360°} + 2 \times \frac{(360° - 60° - 135° - x°)}{360°} + 3 \times \frac{x°}{360°} = 1.5$$
$x = 75$

8. Let n be the total number of the university students in the groups.
So, we have

Number of credit cards	0	1	2	3
Number of university students	$\frac{n}{6}$	$\frac{3n}{8}$	$\frac{n}{4}$	$\frac{5n}{24}$

Since n must be an integer, n is a common multiple of 4, 6, 8 and 24.
Hence the least possible value of n is the least common multiple of 4, 6, 8, 24.
Thus, the least possible value of n is 24.

9. Note that $r = 4$.
$$p = \frac{4+4+4+4+4+4+5+5+6+6+6+9+k}{13} = \frac{61+k}{13}$$
Since $4 \le k \le 9$, we have $5 \le p \le \frac{70}{13}$. (In particular, if $4 < k \le 9$ then $5 < p \le \frac{70}{13}$.)
If $k = 4$, then $q = 4$.
If $5 \le k \le 9$, then $q = 5$.
Therefore, we have $p > q$, $q \ge r$ and $p > r$.
Thus, I and III only.

10. Note that $r = 8$.
$$p = \frac{2+2+2+2+4+5+7+7+8+8+9+10+10+m}{14} = \frac{76+m}{14}$$
Since $5 \le m \le 7$, we have $\frac{81}{14} \le p \le \frac{83}{14}$.
$$q = \frac{7^{\text{th}}\text{ datum} + 8^{\text{th}}\text{ datum}}{2}$$
So, we have $6 \le q \le 7$.
Therefore, we have $p < q$, $p < r$ and $q < r$.
Thus, II and III only.

11. $\frac{(155)(n) + (170)(14)}{14 + n} = 160$
$n = 28$

12. Since the mean of x and y is 4, we have $x + y = 8$.

So, we have

$$\frac{3+6+6+7+13+13+17+18+20+x+y+z}{12} = 10$$

$$\frac{103+(8)+z}{12} = 10$$

$z = 9$

Without loss of generality, assume $x \leq y$.

Then, we have $y \leq 7$.

So, in any cases of possible values of x and y, the sixth datum and the seventh datum are 7 and 9 respectively.

The required median

$= \frac{7+9}{2}$

$= 8$

13. Since the mode of the eight numbers is 3, at least two of x, y and z are 3.

Without loss of generality, assume $x = y = 3$.

Then, we have $\frac{3+3+3+5+7+7+10+z}{8} = 5.5$.

So, we have $z = 6$.

Now, arranging the numbers in ascending order, we have

3 3 3 5 6 7 7 10

The required median

$= \frac{5+6}{2}$

$= 5.5$

14. $\frac{12k-a-2-7-15}{k-4} = 12$

$a = 24$

15. Since the mean of the positive integers is 7, we have

$$\frac{2+7+7+7+9+x+x+y}{8} = 7$$

$2x + y = 24$

Note that the range will be greater than 7 if one of x or y is greater than 9.
Thus, the least possible range of the positive integers is 7.
If $x = 9$ and $y = 6$, then 7 and 9 are the two modes of the positive integers.
So, I is not always true.

If $x = y = 8$, then the median of the positive integers $= \frac{7+8}{2} = 7.5$.

So, III is not always true.

Thus, II only.

16. Since the mode of the data is 5, we have $x = 5$.
The mean of the set of 9 data

$$= \frac{1+3+4+5+5+5+8+10}{9}$$

$= 5$

Since the new mean remains unchanged, the sum of the two removed data = 45 – (5)(7) = 10.
Hence, two data ‘5’ are removed from the set.
Now, arranging the remaining data in ascending order, we have

1 3 4 4 5 8 10

Thus, the new median is 4.

17. Note that the median $= \dfrac{4^{\text{th}} \text{ datum} + 5^{\text{th}} \text{ datum}}{2}$.

Arranging the positive integers (excluding 29 and 41) in ascending order, we have

$x - 12 \quad x - 5 \quad x - 2 \quad x + 2 \quad x + 4 \quad x + 9$

Since $29 < 37$ and $41 > 37$, we may consider the following cases:

(1) If 29 is the 4^{th} datum, then $x - 2 \le 29$ and hence $x \le 31$.

In this case, we must have $\dfrac{29 + (x + 2)}{2} = 37$.

So, we have $x = 43$. However, this contradicts our premise that $x \le 31$.

It is not possible that 29 is the 4^{th} datum.

(2) If 41 is the 5^{th} datum, then $x + 2 \ge 41$ and hence $x \ge 39$.

In this case, we must have $\dfrac{(x - 2) + 41}{2} = 37$.

So, we have $x = 35$. However, this contradicts our premise that $x \ge 39$.

It is not possible that 41 is the 5^{th} datum.

Therefore, we have $\dfrac{(x - 2) + (x + 2)}{2} = 37$.

Hence $x = 37$.

The required mean

$= \dfrac{25 + 29 + 32 + 35 + 39 + 41 + 41 + 46}{8}$

$= 36$

18. $a=\dfrac{2+4+4+4+5+8+9+11+p+q}{10}=\dfrac{47+p+q}{10}$

Without loss of generality, assume $p \le q$.

Since the range of the positive integers is 11, the least possible values of p and q are 1 and 12 respectively.

So, we have $a \ge \dfrac{47+(1)+(12)}{10}=6$.

$b=\dfrac{5^{\text{th}}\text{ datum}+6^{\text{th}}\text{ datum}}{2}$

If $p=1$ and $q=12$, then $b=\dfrac{4+5}{2}=4.5<5$.

So, II is not always true.

If $p=q$, then $p=q=13$. In this case, we have $c=4$.

If $p \ne q$, it is clear that $c=4$.

So, in any cases of possible values of p and q, we have $c=4$.

Thus, I and III only.

19. $\dfrac{16}{16+k+22+10}=\dfrac{1}{5}$

$k=32$

So, the required median $=\dfrac{40^{\text{th}}\text{ datum}+41^{\text{st}}\text{ datum}}{2}$.

Hence the required median is 1.

20. Since the median of the distribution increases after n students joining the group, we have

$16+32 \le 22+10+n$

$n \ge 16$

Thus, the least possible value of n is 16.

21. Since r is a positive integer and the mode of the distribution is 5, we have $1 \le r \le 12$.

Thus, the least possible value and the greatest possible value of r are 1 and 12 respectively.

22. Since the median of the distribution is 3, we have

$11 + r < 9 + 7 + 13$

$r < 18$

$11 + r + 9 > 7 + 13$

$r > 0$

So, we have $0 < r < 18$.

Since r is an integer, we have $1 \le r \le 17$.

Thus, there are 17 possible values for r.

23. Since the mean of the distribution is 3, we have

$$\frac{(1)(11)+(2)(r)+(3)(9)+(4)(7)+(5)(13)}{11+r+9+7+13} = 3$$

$r = 11$

24. Since the mode of the distribution is 43 kg, we have $7 \le a \le 9$ and $8 \le b \le 9$.

$$p = \frac{41+43+43+43+48+52+54+55+56+56+(50+a)+59+60+63+67+67+(60+b)+75+75+(70+b)}{20}$$

$$p = \frac{1137+a+2b}{20}$$

$$p \ge \frac{1137+(7)+2(8)}{20} = 58$$

$$q = \frac{10^{\text{th}}\text{ datum} + 11^{\text{th}}\text{ datum}}{2}$$

$$q = \frac{56+(50+a)}{2}$$

$$q \ge \frac{106+(7)}{2} = 56.5$$

$r = (70 + b) - 41$

$r \ge 29 + (8) = 37$

Thus, I and III only.