

PROBABILITY

Form 5

Vol 10

Part 6B - Probability (with NCR / NPR)

1. (a) The required probability

$$= \left(\frac{3}{6}\right)^{12}$$

$$= \frac{1}{4096}$$

(b) The required probability

$$= \left(\frac{1}{6}\right)^{12} \times (C_3^{12} + P_2^{12} + 12)$$

$$\approx 1.67 \times 10^{-7}$$

(c) The required probability

$$= 1 - \left(\frac{1}{2}\right)^{12}$$

$$= \frac{4095}{4096}$$

(d) The required probability

$$= 1 - \left[\left(\frac{4}{6}\right)^{12} + \left(\frac{3}{6}\right)^{12} - \left(\frac{2}{6}\right)^{12} \right]$$

$$\approx 0.992$$

(e) The required probability

$$= \left[1 - \left(\frac{1}{2}\right)^{12} \times C_6^{12} \right] \times \frac{1}{2}$$

$$= \frac{793}{2048}$$

(f) The required probability

$$= \left(\frac{3}{6}\right)^{12}$$

$$= \frac{1}{4096}$$

2. (a) The required probability

$$= \frac{C_3^{10}}{C_6^{20}}$$

$$= \frac{1}{323}$$

(b) The required probability

$$= \frac{C_1^{10} \times C_4^9 \times 2^4}{C_6^{20}}$$

$$= \frac{168}{323}$$

3. (a) The required probability

$$= \frac{C_8^{20} \times 2^8}{C_8^{40}}$$

$$= \frac{512}{1221}$$

(b) The required probability

$$= \frac{512}{1221} + \frac{C_1^{20} \times C_6^{19} \times 2^6}{C_8^{40}} + \frac{C_2^{20} \times C_4^{18} \times 2^4}{C_8^{40}}$$

$$= \frac{5248}{5291}$$

4. (a) The required probability

$$= 1 - \frac{9-1}{C_2^{10}}$$

$$= \frac{37}{45}$$

(b) The required probability

$$= 1 - \frac{C_2^6}{C_2^{10}}$$

$$= \frac{2}{3}$$

(c) The required probability

$$= \left(1 - \frac{5}{C_2^{10}}\right) \times \frac{1}{2}$$
$$= \frac{4}{9}$$

5. (a) The required probability

$$= \frac{6}{C_2^7}$$
$$= \frac{2}{7}$$

(b) The required probability

$$= 1 - \frac{6}{C_2^7}$$
$$= \frac{5}{7}$$

(c) The required probability

$$= \frac{5}{C_3^7}$$
$$= \frac{1}{7}$$

(d) The required probability

$$= \frac{1}{7} \div 3$$
$$= \frac{1}{21}$$

6. (a) The required probability

$$= \frac{C_4^9 \times C_5^5 + C_3^9 \times C_3^3 \times C_1^6 \times C_5^5 + C_3^9 \times C_4^6 \times C_3^3 \times C_2^2}{C_3^{12} \times C_4^9 \times C_5^5}$$
$$= \frac{3}{44}$$

(b) The required probability

$$= \frac{C_1^9 \times C_4^9 \times C_5^5 + C_2^9 \times C_3^8 \times C_5^5 + C_3^9 \times C_3^7 \times C_4^4}{C_3^{12} \times C_4^9 \times C_5^5}$$
$$= \frac{29}{132}$$

Part 7 - Expected Value

1. B

1. The possible outcomes of selecting 3 banknotes are: $\{\$10, \$10, \$10\}$, $\{\$10, \$10, \$20\}$, $\{\$10, \$20, \$20\}$ and $\{\$20, \$20, \$20\}$;

The corresponding amounts of the 3 banknotes are \$30, \$40, \$50 and \$60 respectively; while the

corresponding probabilities are $\frac{C_3^7}{C_3^{13}}$, $\frac{C_2^7 \times C_1^6}{C_3^{13}}$, $\frac{C_1^7 \times C_2^6}{C_3^{13}}$ and $\frac{C_3^6}{C_3^{13}}$ respectively.

The expected amount of the 3 selected banknotes

$$= 30 \times \frac{35}{286} + 40 \times \frac{63}{143} + 50 \times \frac{105}{286} + 60 \times \frac{10}{143}$$

$$\approx 43.84615385$$

$$\approx \$43.8$$

2. The expected amount from the travelling insurance

$$= (300)(0.2) + (1000)(0.1)$$

$$= \$160$$

$$< \$200$$

Thus, Maggie should not buy the travelling insurance.

3. (a) The expected number of times that Mary travels by bus on a school day

$$= (2)(0.15) + (1)(0.2 + 0.2)$$

$$= 0.7$$

- (b) The expected amount spent by Mary on the transport on a school day

$$= (3.5 + 2.8)(0.15) + (3.5)(0.2) + (2.8)(0.2)$$

$$= \$2.205$$

4. (a) The required probability

$$= \frac{1}{C_3^5}$$

$$= \frac{1}{10}$$

- (b) The expected number of black balls drawn

$$= (3) \left(\frac{1}{10} \right) + (2) \left(\frac{C_2^3 \times C_1^2}{C_3^5} \right) + (1) \left(\frac{C_1^3 \times C_2^2}{C_3^5} \right)$$

$$= 1.8$$

5. The expected value of the fare for Tom to return home after school

$$= (3.2)(0.2) + (4)(0.55) + (6.8)(0.25)$$

$$= \$4.54$$

6. (a) (i) The required probability

$$= \frac{4}{6} \times \frac{3}{4}$$

$$= \frac{1}{2}$$

(ii) The required probability

$$= \frac{4}{6} \times \frac{1}{4} + \frac{2}{6} \times \frac{1}{4}$$

$$= \frac{1}{4}$$

(iii) The possible amounts get by Jenny are: \$20, \$30, \$60 and \$70.

The corresponding probabilities are: $\frac{1}{2}$, $\frac{2}{6} \times \frac{3}{4}$, $\frac{4}{6} \times \frac{1}{4}$ and $\frac{2}{6} \times \frac{1}{4}$ respectively.

The expected amount that Jenny can get

$$= (20)\left(\frac{1}{2}\right) + (30)\left(\frac{1}{4}\right) + (60)\left(\frac{1}{6}\right) + (70)\left(\frac{1}{12}\right)$$

$$= \$\frac{100}{3}$$

(b) (i) The required probability

$$= \frac{4}{6} \times \frac{3}{5} \times \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{1}{5}$$

(ii) The possible amounts get by Jenny are: \$40, \$50, \$60, \$80, \$90 and \$100.

The corresponding probabilities are: $\frac{1}{5}$, $\frac{4}{6} \times \frac{2}{5} \times 2 \times \frac{3}{4} \times \frac{2}{3}$, $\frac{2}{6} \times \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}$,

$$\frac{4}{6} \times \frac{3}{5} \times \frac{3}{4} \times \frac{1}{3} \times 2, \frac{4}{6} \times \frac{2}{5} \times 2 \times \frac{3}{4} \times \frac{1}{3} \times 2 \text{ and } \frac{2}{6} \times \frac{1}{5} \times \frac{3}{4} \times \frac{1}{3} \times 2.$$

The expected amount that Jenny can get

$$= (40)\left(\frac{1}{5}\right) + (50)\left(\frac{4}{15}\right) + (60)\left(\frac{1}{30}\right) + (80)\left(\frac{1}{5}\right) + (90)\left(\frac{4}{15}\right) + (100)\left(\frac{1}{30}\right)$$

$$= \$\frac{200}{3}$$

Part 8 - Conditional Probability

1. A

2. C

1. The required probability

$$= \frac{(60\%)(2\%)}{(60\%)(2\%) + (40\%)(5\%)}$$
$$= \frac{3}{8}$$

2. The table below shows the possible outcomes that the sum of the two number thrown is an even number:

	1	2	3	4	5	6
1	2		4		6	
2		4		6		8
3	4		6		8	
4		6		8		10
5	6		8		10	
6		8		10		12

Also, the possible outcomes that at least one number thrown is 4 are highlighted as shown in the above table.

Thus, the required probability is $\frac{5}{18}$.

3. (a) The required probability

$$= 1 - (0.2 + 0.25)$$
$$= 0.55$$

(b) The required probability

$$= \frac{0.4}{0.2 + 0.4 + 0.25}$$
$$= \frac{8}{17}$$

4. The required probability

$$= \frac{\left(\frac{5}{11}\right)\left(\frac{2}{10}\right)\left(\frac{4}{9}\right) \times 2}{\left(\frac{9}{11}\right)\left(\frac{2}{10}\right)\left(\frac{9}{9}\right) + \left(\frac{2}{11}\right)\left(\frac{1}{10}\right)\left(\frac{9}{9}\right)}$$
$$= \frac{4}{9}$$

5. (a) The required probability

$$= \left(\frac{1}{3}\right)\left(1 - \frac{1}{6}\right) + \left(1 - \frac{1}{3}\right)\left(\frac{1}{6}\right)$$

$$= \frac{7}{18}$$

(b) The required probability

$$= \frac{\left(\frac{1}{3}\right)\left(1 - \frac{1}{6}\right)}{\frac{7}{18}}$$

$$= \frac{5}{7}$$

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