PROBABILITY

Form 5

Vol 10

Part 5 - Until win

1. (a) Probability of getting a number greater than 4 $= \frac{2}{6} = \frac{1}{3}$

Probability of getting an even number $= \frac{3}{6} = \frac{1}{2}$

The required probability

$$= \frac{1}{2} + \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(\frac{1}{2}\right) + \cdots$$

$$= \frac{\frac{1}{2}}{1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)}$$

$$= \frac{3}{4}$$

(b) The required probability

$$= \left(1-\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(\frac{1}{2}\right) + \cdots$$

$$= \frac{\left(1-\frac{1}{3}\right)\left(\frac{1}{2}\right)}{1-\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)}$$

$$= \frac{1}{2}$$

2. The required probability

$$= \left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{3}{7}\right)\left(\frac{4}{6}\right)$$

$$= \frac{5}{63}$$

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3. (a) There are 3 possible events occurring within the first two possible draws.

Event (1): a red ball is drawn in the first draw from box A. (The player wins the game.)

Probability that event (1) occurs $= \dfrac{1}{2}$

Event (2): a green ball is drawn in the first draw from box A and a red ball is drawn in the following draw from box B. (The player must win the game in the next drawn in box A.)

Probability that event (2) occurs $= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{3}\right)$

Event (3): a green ball is drawn in the first draw from box A and a green ball is drawn in the following draw from box B.

Probability that event (3) occurs $= \left(\dfrac{1}{2}\right)\left(1-\dfrac{1}{3}\right) = \left(\dfrac{1}{2}\right)\left(\dfrac{2}{3}\right)$

If event (3) occurs, then the winning situation will be the same as the start of the game.

Hence, we have $p = \dfrac{1}{2} + \left(\dfrac{1}{2}\right)\left(\dfrac{1}{3}\right) + \left(\dfrac{1}{2}\right)\left(\dfrac{2}{3}\right)p$.

(b) The required probability

$= \dfrac{1}{2} + \left(\dfrac{1}{2}\right)\left(\dfrac{1}{3}\right) + \left(\dfrac{1}{2}\right)\left(\dfrac{2}{3}\right)\left(\dfrac{1}{2}\right)$

$= \dfrac{5}{6}$

4. The required probability

$= \left(1-\dfrac{2}{9}\right)\left(\dfrac{1}{6}\right) + \left(1-\dfrac{2}{9}\right)\left(1-\dfrac{1}{6}\right)\left(1-\dfrac{2}{9}\right)\left(\dfrac{1}{6}\right) + \left(1-\dfrac{2}{9}\right)\left(1-\dfrac{1}{6}\right)\left(1-\dfrac{2}{9}\right)\left(1-\dfrac{1}{6}\right)\left(1-\dfrac{2}{9}\right)\left(\dfrac{1}{6}\right) + \cdots$

$= \dfrac{\left(1-\dfrac{2}{9}\right)\left(\dfrac{1}{6}\right)}{1-\left(1-\dfrac{2}{9}\right)\left(1-\dfrac{1}{6}\right)}$

$= \dfrac{7}{19}$

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5. (a) The required probability

$= \frac{3}{8}$

(b) The required probability

$= \left(1-\frac{3}{8}\right)\left(\frac{3}{8}\right) + \left(1-\frac{3}{8}\right)^3\left(\frac{3}{8}\right) + \left(1-\frac{3}{8}\right)^5\left(\frac{3}{8}\right) + \left(1-\frac{3}{8}\right)^7\left(\frac{3}{8}\right)$

$= \frac{6302535}{16777216}$ | r.t. 0.376

(c) The required probability

$= \left(1-\frac{3}{8}\right)\left(\frac{3}{8}\right) + \left(1-\frac{3}{8}\right)^3\left(\frac{3}{8}\right) + \left(1-\frac{3}{8}\right)^5\left(\frac{3}{8}\right) + \cdots$

$= \frac{\left(1-\frac{3}{8}\right)\left(\frac{3}{8}\right)}{1-\left(1-\frac{3}{8}\right)^2}$

$= \frac{5}{13}$

Part 6A - Probability (with NCR / NPR)

1. (a) The required probability

$= \frac{C_2^5 \times C_2^6 \times C_3^4}{C_7^{15}}$

$= \frac{40}{429}$

(b) The required probability

$= \frac{C_3^{11} \times C_4^4}{C_7^{15}} + \frac{40}{429}$

$= \frac{17}{143}$

2. The required probability

$= \frac{C_5^{12} - (C_5^6 + C_5^6)}{C_5^{19}} + \frac{C_5^{13} - (C_5^6 + C_5^7)}{C_5^{19}} \times 2$

$= \frac{275}{969}$

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3. (a) The required probability

$$= 1 - \frac{C_4^{10}}{C_4^{18}}$$

$$= \frac{95}{102}$$

(b) The required probability

$$= \frac{C_4^{10}}{C_4^{18}} + \frac{C_3^{10} \times C_1^{8}}{C_4^{18}}$$

$$= \frac{13}{34}$$

4. The required probability

$$= 1 - \frac{C_4^{12}}{C_4^{15}}$$

$$= \frac{58}{91}$$

5. (a) The required probability

$$= 1 - \frac{C_4^{7}}{C_4^{12}}$$

$$= \frac{92}{99}$$

(b) The required probability

$$= 1 - \left(\frac{C_4^{7}}{C_4^{12}} + \frac{C_4^{8}}{C_4^{12}} \right) \quad \text{| or} \quad \frac{C_3^5 \times C_1^4 + C_2^5 \times (C_2^4 + C_1^4 \times C_1^3) + C_1^5 \times (C_3^4 + C_2^4 \times C_1^3 + C_1^4 \times C_2^3)}{C_4^{12}}$$

$$= \frac{26}{33}$$

(c) The required probability

$$= \frac{C_1^5 \times C_1^4 \times C_1^3 \times C_1^9 \times \frac{1}{2!}}{C_4^{12}}$$

$$= \frac{6}{11}$$

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6.	B	7.	A

6. The required probability

$$=\frac{C_1^5\times C_2^4+C_3^5}{C_3^9}$$

$$=\frac{10}{21}$$

7. The required probability

$$=\frac{C_7^{10}\times 2}{10!}$$

$$=\frac{1}{15120}$$

8. (a) The required probability

$$=\frac{9}{C_3^{11}}$$

$$=\frac{3}{55}$$

(b) The required probability

$$=1-\frac{3}{55}-\frac{C_3^9}{C_3^{11}} \quad | \text{ or } \quad \frac{2\times 8+8\times 7}{C_3^{11}}$$

$$=\frac{24}{55}$$

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9. (a) The required probability

$= \frac{C_3^7}{C_3^{25}}$

$= \frac{7}{460}$

(b) The required probability

$= \frac{C_3^{10} + C_3^7 + C_3^8}{C_3^{25}}$

$= \frac{211}{2300}$

(c) The required probability

$= \frac{8 \times 7 \times 16 + 8 \times 7 \times 7 + 16 \times 7 \times 8}{24 \times 7 \times 23}$ | or $\frac{C_2^8 + 8 \times 16}{C_2^{24}}$

$= \frac{13}{23}$

(d) The required probability

$= \frac{10 \times 15 \times 9 + 7 \times 18 \times 6 + 8 \times 17 \times 7 + 10 \times 7 \times 8 \times 3!}{P_3^{25}}$

$= \frac{3209}{6900}$

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