CONGRUENT AND SIMILAR TRIANGLES

Form 1 Regular Course

Vol 9

Part 1 – Proving congruent triangles

1. $\Delta ABC \cong \Delta EDF$ (SSS)

2. $\Delta ABC \cong \Delta DEF$ (SAS)

3. $\Delta PQR \cong \Delta ZYX$ (ASA) (or AAS)

4. $\Delta ABC \cong \Delta QRP$ (RHS)

5. $\Delta ABC \cong \Delta YZX$ (RHS)

6. $\angle ABD = \angle ACD$ (given)
   $\angle BAD = \angle CAD$ (given)
   $AD = AD$ (common)
   Thus, $\Delta ABD \cong \Delta ACD$ (AAS).

7. (a) $AB = CD$ (given)
   $BD = DB$ (common)
   $AD = CB$ (given)
   Thus, $\Delta ABD \cong \Delta CDB$ (SSS).

   (b) From (a), $\angle ADB = \angle CBD$ (corr. $\angle$s, $\cong\Delta$s)
   $\angle DAN = \angle BCM$ (given)
   $AD = CB$ (given)
   Thus, $\Delta ADN \cong \Delta CBM$ (ASA).

8. A

The conditions provided in the two triangles is ASS, which is not valid for proving congruence.

B: $\Delta ABC \cong \Delta FED$ (SAS)

C: $\angle ACB = 180° - 85° - 47° = 48°$ ($\angle$ sum of Δ)

$\Delta ABC \cong \Delta FED$ (ASA)

D. $\Delta ABC \cong \Delta FED$ (RHS)

9. D

$\Delta ABC \cong \Delta FED$ (RHS)

A: The conditions provided in the two triangles is ASS.

B: The conditions provided in the two triangles is AAA.

C: The conditions provided in ΔABC and ΔDEF are SAS and RHS respectively.

10. $AB = BA$ (common)

$AD = BC$ (given)

$\angle ADB = \angle BCA = 90°$ (given)

Thus, $\Delta ABD \cong \Delta BAC$ (RHS)

11. $AE = AD$ (given)

$\angle BAE = \angle CAD$ (common)

Since $BD = CE$ (given),

$AB = AD + BD = AE + CE = AC$

Thus, $\Delta ABE \cong \Delta ACD$ (SAS)

Part 2 – Using congruent triangles

1. $x = QR = CB = 7$ cm (corr. sides, ≅Δs)
   $y = PR = AB = 4$ cm (corr. sides, ≅Δs)
   $z = PQ = AC = 8$ cm (corr. sides, ≅Δs)

2. $x + 3 = AB = QP = 7$ (corr. sides, ≅Δs)
   $x = 4$
   $2y = AC = QR = 8$ (corr. sides, ≅Δs)
   $y = 4$
   $z = \angle BAC = \angle PQR$ (corr. ∠s, ≅Δs)
   $z = 180° - 58° - 62° = 60°$ (∠ sum of Δ)

3. $(x - 2)$ cm $= XY = PQ = 18$ cm (corr. sides, ≅Δs)
   $x = 20$
   $(2y + 7)$ cm $= YZ = QR = 9$ cm (corr. sides, ≅Δs)
   $y = 1$
   $z = PR = XZ = x + y = 21$ (corr. sides, ≅Δs)

4. $AC = DC = 7$ cm (corr. sides, ≅Δs)
   $BC = EC = DE - CD = 3$ cm (corr. sides, ≅Δs)
   $AB = AC - BC = 4$ cm

5. $TR = QR = 4$ cm (corr. sides, ≅Δs)
   $2a + 3 + 4 = PR = SR = 6a - 1$ (corr. sides, ≅Δs)
   $a = 2$

6. $\angle BAD = \angle CAD = 50° \div 2 = 25°$ (corr. ∠s, ≅Δs)
   $\angle ACD = \angle ABD = 30°$ (corr. ∠s, ≅Δs)
   Considering ΔACD,
   $x = 25° + 30° = 55°$ (ext. ∠ of Δ)

7. A

I may not be true.

$AB = ZY$ (corr. sides, $\cong\Delta$s), while ZY may not equal XY.

II must be true.

$AB = YZ$ (corr. sides, $\cong\Delta$s)

$BC = XY$ (corr. sides, $\cong\Delta$s)

Thus, $AB + BC = XY + YZ$.

III may not be true.

$\angle ABC = \angle XYZ$ (corr. $\angle$s, $\cong\Delta$s)

$\angle BCA = \angle YXZ$ (corr. $\angle$s, $\cong\Delta$s)

Thus, $\angle ABC + \angle BCA = \angle XYZ + \angle YXZ$, while $\angle YXZ$ may not equal $\angle YZX$.

Part 3A – Prove first (A)

1. (a) $BK = DK$ (given)
$\angle AKB = \angle AKD$ (given)
$AK = AK$ (common)
Thus, $\Delta ABK \cong \Delta ADK$ (SAS).
(b) $AB = AD$ (corr. sides, $\cong\Delta$s)
$\angle BAK = \angle DAK$ (corr. $\angle$s, $\cong\Delta$s)
$AC = AC$ (common)
Thus, $\Delta ABC \cong \Delta ADC$ (SAS).
(c) $\angle ADK = \angle ABK = 180° - 90° - 48° = 42°$
$\angle ABC = \angle ADC = 63° + 42° = 105°$

2. (a) $\angle ABC = \angle CDE = 108°$ (given)
$\angle ACB = 180° - 25° - 108° = 47°$ ($\angle$ sum of Δ)
$= \angle CED$
$AC = CE$ (given)
Thus, $\Delta ABC \cong \Delta CDE$ (AAS)
(b) $\angle DCE = \angle BAC = 25°$ (corr. $\angle$s, $\cong\Delta$s)
$x = 180° - \angle ACB - \angle DCE = 108°$ (adj. $\angle$s on st. line)
$x - y = \angle DCE = 25°$
$y = 108° - 25° = 83°$

3. (a) $\angle ABD = \angle ECD$ (given)
$\angle CDE = 360° - 112° - 136° = 112°$ ($\angle$s at a pt.)
$= \angle BDA$
$AD = ED$ (given)
Thus, $\Delta ABD \cong \Delta ECD$ (AAS).
(b) $\angle ECD = \angle CEB - \angle CDE$ (ext. $\angle$ of Δ)
$\angle ECD = 136° - 112° = 24°$
$\angle ABD = \angle ECD = 24°$
(c) $BD = CD = 34$ cm (corr. sides, $\cong\Delta$s)
$DE = DA = 20$ cm (corr. sides, $\cong\Delta$s)
$BE = BD - DA = 14$ cm