JUNIOR REVISION

Form 6
Vol 9

Part 5 – Estimation and Errors

1. D	2. B	3. B	4. D	5. C	6. A	7. A
8. C						

1. If x is correct to 2 decimal places, then $x = 0.03 \neq 0.034$.
   If x is correct to 2 significant figures, then $x = 0.035 \neq 0.034$.
   If x is correct to 3 decimal places, then $x = 0.035 \neq 0.0346$.
   If x is correct to 3 significant figures, then $x = 0.0346$.
   Thus, D is the desired answer.

2. Note that $3.465 \leq x < 3.475$ and $2.05 \leq y < 2.15$.
   So, we have $2(3.465) - 2.15 < 2x - y < 2(3.475) - 2.05$.
   Thus, we have $4.78 < 2x - y < 4.9$.

3. If 0.0689 is rounded down to 2 significant figures, then the result is 0.068.
   The percentage error
   $= \frac{0.0689 - 0.068}{0.0689} \times 100\%$
   $\approx 1.31\%$

4. Let x kg be the actual weight of Peter.
   The lower limit of the weight of Peter = 80(1 – 5%) = 76 kg
   The upper limit of the weight of Peter = 80(1 + 5%) = 84 kg
   So, we have $76 \leq x < 84$.
   Hence, it is not possible that the actual weight of Peter is 84 kg.

5. The measured duration of the second part of the performance = 120 – 56 = 64 seconds
The maximum absolute error in calculating the duration of the second part of the performance.

$=\left[(120+\frac{1}{2})-(56-\frac{2}{2})\right]-64$

= 1.5 seconds

The required relative error

$=\frac{1.5}{64}$

$=\frac{3}{128}$

6. The lower limit of the length of the string = 25(1 – 20%) = 20 m
The upper limit of the length of the string = 25(1 + 20%) = 30 m

The lower limit of the length of $A = 8-\frac{2}{2} = 7$ m

The upper limit of the length of $A = 8+\frac{2}{2} = 9$ m

So, we have $20-9 < x < 30-7$.
Thus, we have $11 < x < 23$.

7. The upper limit of the volume of water in bottle $A = 200+\frac{40}{2} = 220$ mL

The upper limit of the volume of water in bottle $B = 350+\frac{50}{2} = 375$ mL

The lower limit of the volume of water in each cup = $10-\frac{1}{2} = 9.5$ mL

So, we have $n < \frac{220+375}{9.5} \approx 62.63157895$.

Thus, the greatest possible value of n is 62.

8. Maximum absolute error of the measurements = $\frac{1}{2} = 0.5$ cm

The lower limit of the length of $DE = (20-0.5)-(10+0.5)+(10-0.5) = 18.5$ cm
The upper limit of the length of $DE = (20+0.5)-(10-0.5)+(10+0.5) = 21.5$ cm
So, we have $(20-0.5+18.5)(10-0.5) < x < (20+0.5+21.5)(10+0.5)$.
Thus, we have $361 < x < 441$.

9. (a) 5 116
(b) 5 116.74
(c) 5 200

10. (a) The lower limit of the length of the rope $= 150 - \frac{1}{2} = 149.5$ cm

The upper limit of the length of the rope $= 150 + \frac{1}{2} = 150.5$ cm

The lower limit of the length of $A = 76 - \frac{2}{2} = 75$ cm

The upper limit of the length of $A = 76 + \frac{2}{2} = 77$ cm

The lower limit of the length of $B = 149.5 - 77 = 72.5$ cm

The upper limit of the length of $B = 150.5 - 75 = 75.5$ cm

(b) The measured length of $B = 150 - 76 = 74$ cm

The maximum absolute error in calculating the length of $B = 75.5 - 74 = 1.5$ cm

The required relative error

$= \frac{1.5}{74}$

$= \frac{3}{148}$ r.t. 0.0203

11. (a) The least possible volume of a box of milk

$= 946 - \frac{1}{2}$

$= 945.5$ mL

(b) The upper limit of the volume of a glass of milk that Peter has to drink each day

$= 310 + \frac{10}{2}$

$= 315$ mL

The volume of a glass of milk that Peter can drink each day in the coming 3 weeks

$\geq \frac{945.5 \times 7}{7 \times 3}$

$= 315\frac{1}{6}$ mL

> 315 mL

Thus, it is possible for Peter to have a glass of milk every day in the coming 3 weeks.

12. (a) Maximum absolute error of the measurements $= \frac{1}{2} = 0.5$ cm

The lower limit of the area of the hexagon
$= (12 - 0.5)(8 - 0.5) - (9 + 0.5)(4 + 0.5)$
$= 43.5\ \text{cm}^2$

The upper limit of the area of the hexagon
$= (12 + 0.5)(8 + 0.5) - (9 - 0.5)(4 - 0.5)$
$= 76.5\ \text{cm}^2$

(b) The lower limit of the perimeter of the hexagon
$= 2[(12 - 0.5) + (8 - 0.5)]$
$= 38$ cm

The upper limit of the perimeter of the hexagon
$= 2[(12 + 0.5) + (8 + 0.5)]$
$= 42$ cm

Part 6 – Numerical System

1. C	2. C	3. D	4. C	5. B

1. $9 \times 2^{10} + 2^8 + 5 \times 2^5 + 2 \times 2^3 - 2^3$
$= (2^3 + 1) \times 2^{10} + 2^8 + (2^2 + 1) \times 2^5 + (2 - 1) \times 2^3$
$= 2^{13} + 2^{10} + 2^8 + 2^7 + 2^5 + 2^3$
Thus, we have $9 \times 2^{10} + 2^8 + 5 \times 2^5 + 2 \times 2^3 - 2^3 = 10010110101000_2$.

2. $58 \times 16^9 + 12 \times 16^4 + 122$
$= (3 \times 16 + 10) \times 16^9 + 12 \times 16^4 + (7 \times 16 + 10)$
$= 3 \times 16^{10} + 10 \times 16^9 + 12 \times 16^4 + 7 \times 16^1 + 10 \times 16^0$
Thus, we have $58 \times 16^9 + 12 \times 16^4 + 122 = 3A0000C007A_{16}$.

3. $2^{33} + 2^{30} + 2^{11} - 2^9$
$= 2 \times 2^{32} + 4 \times 2^{28} + (8 - 2) \times 2^8$
$= 2 \times 16^8 + 4 \times 16^7 + 6 \times 16^2$
Thus, $2^{33} + 2^{30} + 2^{11} - 2^9 = 240000600_{16}$.

4. Note that $8^{19} = 2^{57} = 2 \times 16^{14}$.
$3 \times 8^{19} + 2\,019$
$= 3 \times (2 \times 16^{14}) + (7 \times 16^2 + 14 \times 16 + 3)$
$= 6 \times 16^{14} + 7 \times 16^2 + 14 \times 16 + 3$
Thus, we have $3 \times 8^{19} + 2\,019 = 6000000000007E3_{16}$.

5. $D689_{16}$

$= 13 \times 16^3 + 6 \times 16^2 + 8 \times 16 + 9$

$= (2^3 + 2^2 + 1) \times 2^{12} + (2^2 + 2) \times 2^8 + 2^3 \times 2^4 + 2^3 + 1$

$= 2^{15} + 2^{14} + 2^{12} + 2^{10} + 2^9 + 2^7 + 2^3 + 1$

Thus, we have $D689_{16} = 1101011010001001_2$.

Part 7 – Polar coordinate

1.	C	2.	B	3.	C

1. First we have $B(3\sqrt{3}, -3)$ and $C(-3\sqrt{3}, 3)$.

Then, the coordinates of M are $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$.

Note that AC is horizontal.

The area of $\triangle AMC$

$= \frac{(3\sqrt{3} + 3\sqrt{3})\left(3 + \frac{3}{2}\right)}{2}$

$= \frac{27\sqrt{3}}{2}$

≈ 23

2. First we have $B(3, -a)$ and $C(3, 8 - a)$.

Note that BC is vertical.

Then by considering the area of $\triangle ABC$, we have

$\frac{(8)(a-3)}{2} = 16$

$a = 7$

3. Denote the origin by O.

Then, we have $OA = OB = OC$ and $\angle AOB = 90°$.

$\angle OBA = \frac{180° - 90°}{2} = 45°$

$\tan \angle OCB = \sqrt{3}$

$\angle OCB = 60°$

$\angle OBC = \angle OCB = 60°$

Thus, we have $\angle ABC = 45° + 60° = 105°$.

4. (a) $C(k, 4)$

$D(-k, -3)$

(b) Note that $BC \perp BD$.

By considering the area of $\triangle BCD$, we have

$$\frac{(4+3)(k+k)}{2} = 28$$

$k = 4$

5. (a) Denote the pole by O. Then, we have $\angle AOB = 90°$.

Note that $AB : OA = \sqrt{2} : 1$.

Thus, $AB = 8\sqrt{2}$.

(b) Let M be the mid-point of AB.

Since $OA = OB$, we have $OM \perp AB$ and OM bisects $\angle AOB$.

$\angle BOC = 73° + (360° - 298°) = 135°$

$$\angle MOC = 135° + \frac{90°}{2} = 180°$$

Thus, MOC is a straight line.

Note that $OM : OA = 1 : \sqrt{2}$.

$$CM = 3 + \frac{8}{\sqrt{2}} = 3 + 4\sqrt{2}$$

The area of $\triangle ABC$

$$= \frac{(8\sqrt{2})(3+4\sqrt{2})}{2}$$

$= 12\sqrt{2} + 32$

6. Denote the pole by O.

Since $\angle BOC = 180°$, BC is a straight line.

$\angle AOB = 140° - 80° = 60°$

The perpendicular distance from A to $BC = 4 \sin 60° = 2\sqrt{3}$

The area of $\triangle ABC$

$$= \frac{(8+4)(2\sqrt{3})}{2}$$

$= 12\sqrt{3}$

7. (a) $Q(10, 230°)$

$R(10, 350°)$

(b) Let $M(r, \theta)$ be the polar coordinates of the mid-point of QR. Denote the pole by O.

Since $OQ = OR$, we have $OM \perp QR$ and OM bisects $\angle QOR$.

$$\angle QOM = \frac{120°}{2} = 60°$$

So, we have $r = 10\cos 60° = 5$ and $\theta = 230° + 60° = 290°$.

Thus, the polar coordinates of M are $(5, 290°)$.

8. (a) $B(6, 85°)$

$C(6, 145°)$

(b) Note that $\angle AOB = \angle BOC = 60°$ and $OA = OB = OC$.

So, we have $AB = BC = 6$ and $AC = (2)(6\sin 60°) = 6\sqrt{3}$.

The perimeter of $\triangle ABC$

$= 6 + 6 + 6\sqrt{3}$

$= 12 + 6\sqrt{3}$