PERMUTATION AND COMBINATION

Form 5

Vol 09

Part 3B – Combination

1.	B	2.	C	3.	B	4.	A	5.	C	6.	D	7.	B
8.	B	9.	B	10.	B	11.	D	12.	C	13.	C	14.	C
15.	A	16.	B	17.	C	18.	C						

1. The required number

$= \dfrac{C_3^{18} \times C_3^{15} \times C_3^{12} \times C_3^9 \times C_3^6 \times C_3^3}{6!}$

$= 190\ 590\ 400$

2. The required number

$= 190\ 590\ 400 - \dfrac{C_2^2 \times C_1^{16} \times C_3^{15} \times C_3^{12} \times C_3^9 \times C_3^6 \times C_3^3}{5!}$

$= 168\ 168\ 000$

3. The required number

$= C_2^{10} \times C_4^8 \times C_4^4$

$= 3\ 150$

4. The required number

$= \dfrac{3\,150}{2!}$

$= 1\ 575$

5. The required number

$= 1\ 575 \times 3!$

$= 9\ 450$

6. The required number

$= C_3^{12} \times C_3^9 \times C_3^6 \times C_3^3$

$= 369\ 600$

7. The required number

$= \dfrac{369\,600}{4!}$

$= 15\,400$

8. The required number

$= C_4^4 \times C_2^6 \times 6!$

$= 10\,800$

9. The required number

$= C_4^4 \times 4! \times C_2^6 \times 2! \times 2 + C_3^4 \times 3! \times C_3^6 \times 3! \times 2 + C_2^4 \times 2! \times C_4^6 \times 4! \times 2 + C_1^4 \times C_5^6 \times 5! \times 2 + C_6^6 \times 6!$

$= 22\,320$

10. The required number of arrangements

$= (C_4^8 \times C_2^8 + C_5^8 \times C_1^8 + C_6^8) \times 6!$

$= 1\,753\,920$

11. The required number of arrangements

$= C_3^8 \times C_3^8 \times 3! \times P_3^4 + C_4^8 \times C_2^8 \times 4! \times P_2^5 + C_5^8 \times C_1^8 \times 6! + C_6^8 \times 6!$

$= 1\,735\,104$

12. The required number of ways

$= C_4^7 \times 4!$

$= 840$

13. The required number of ways

$= C_2^2 \times C_2^5 \times 4!$

$= 240$

14. The required number of ways

$= C_2^2 \times C_2^5 \times 3!$

$= 60$

15. The required number of ways

$= 240 - (60 + C_2^2 \times C_2^5 \times 3! - C_2^2 \times C_2^5 \times 2!)$

$= 140$

16. The required number

$= C_3^6 \times C_3^5 \times 3! \times 4!$

$= 28\ 800$

17. The required number

$= C_2^5 \times C_3^5 \times C_1^5 \times 5!$

$= 60\ 000$

18. The required number

$= C_2^5 \times C_2^4 \times 4! \times P_2^5 + C_2^5 \times C_3^4 \times 6! + C_3^5 \times C_2^4 \times 6! + C_3^5 \times C_3^4 \times 6!$ | or $C_3^6 \times C_3^5 \times 6! - C_2^5 \times C_2^4 \times 2! \times 5!$

$= 129\ 600$

Part 4 – Game

1.	C	2.	D	3.	B	4.	B	5.	B	6.	C	7.	D

1. Finish with 5 times (5 x 2 stairs) = 1

Finish with 6 times (4 x 2 stairs and 2 x 1 stair) = C_4^6

Finish with 7 times (3 x 2 stairs and 4 x 1 stair) = C_3^7

Finish with 8 times (2 x 2 stairs and 6 x 1 stair) = C_2^8

Finish with 9 times (1 x 2 stairs and 8 x 1 stair) = C_1^9

Finish with 10 times (10 x 1 stair) = 1

The required number

$= 1 + C_1^9 + C_2^8 + C_3^7 + C_4^6 + 1$

$= 89$

2. Finish with 7 times (6 x 3 stairs and 1 x 2 stairs) = C_6^7

   Finish with 8 times (4 x 3 stairs and 4 x 2 stairs) = C_4^8

   Finish with 9 times (2 x 3 stairs and 7 x 2 stairs) = C_2^9

   Finish with 10 times (10 x 2 stairs) = 1

   The required number

   $=1 + C_2^9 + C_4^8 + C_6^7$

   = 114

3. Each path is an 8-step path consisting of 3 N's and 5 E's.

   The required number

   $= C_3^8 \times C_5^5$

   = 56

4. From A to C, 4-step path consisting of 2 N's and 2 E's.

   From C to B, 4-step path consisting of 1 N's and 3 E's.

   The required number

   $= C_2^4 \times C_2^2 \times C_1^4 \times C_3^3$

   = 24

5. The required number

   $= C_2^{15} + C_1^{15} \times C_1^5 + 1$ | or $C_2^{20} - C_2^5 + 1$

   = 181

6. The required number

   $= C_3^{20} - C_3^5$ | or $C_3^{15} + C_2^{15} \times C_1^5 + C_1^{15} \times C_2^5$

   = 1 130

7. Number of diagonals in a regular n-sided polygon $= C_2^n - n$

So, we have

$$\frac{n!}{(n-2)!\times 2!} - n = 8n + 10$$

$$\frac{n(n-1)}{2} - n = 8n + 10$$

$n^2 - 19n - 20 = 0$

$(n + 1)(n - 20)=0$

$n = -1$ (rejected) or $n = 20$