JUNIOR REVISION

Form 6
Vol 9

Part 1 – Mensuration (3D)

1. B	2. D	3. B	4. C	5. A	6. C	7. D
8. B	9. C	10. B				

1. The height of the smaller pyramid *KEFGH*

$$= \sqrt{26^2 - \left(\frac{10\sqrt{2}}{\sqrt{2}}\right)^2}$$

$= 24$ cm

Note that $KA : KE = 3 : 2$.

The volume of the frustum *ABFEHDCG*

$$= \frac{1}{3}(10\sqrt{2})^2(24)\left(\frac{3^3 - 2^3}{2^3}\right)$$

$= 3800 \text{ cm}^3$

2. The radius of the upper face of the frustum *ABCD*

$= \frac{16\pi}{2\pi}$

$= 8$ cm

The radius of the base of the frustum *ABCD*

$= \frac{48\pi}{2\pi}$

$= 24$ cm

The height of the frustum *ABCD*

$= \sqrt{34^2 - (24 - 8)^2}$

$= 30$ cm

The volume of the frustum

$= \frac{\pi}{3}(30)[24^2 + (24)(8) + 8^2]$

$= 8\ 320\pi$

$\approx 26\ 100$ cm^3

3. Let h cm be the height of the circular cone.

$\frac{4}{3}\pi(4)^3 + \frac{4}{3}\pi(5)^3 = \frac{1}{3}\pi(6)^2 h$

$h = 21$

The slant height of the circular cone

$= \sqrt{6^2 + 21^2}$

$= 3\sqrt{53}$ cm

The curved surface area of the circular cone

$= \pi(6)(3\sqrt{53})$

≈ 412 cm^2

4. $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h + \pi\left(\frac{r}{2}\right)^2 h$

$16r = 7h$

Thus, we have $r : h = 7 : 16$.

5. Let r and h be the base radius and the height of the cylinder respectively.
Then the radius of the hemisphere is r.

$2(2\pi rh) = 5(2\pi r^2)$

$\frac{h}{r} = \frac{5}{2}$

$\frac{\text{volume of the cylinder}}{\text{volume of the hemisphere}}$

$= \frac{\pi r^2 h}{\frac{2}{3}\pi r^3}$

$= \frac{3h}{2r}$

$= \frac{3}{2}\left(\frac{5}{2}\right)$

$= \frac{15}{4}$

6. $\frac{1}{3}\pi r^2 h = \pi r^2(2r)$

$h = 6r$

$\frac{\text{curved surface area of the cone}}{\text{curved surface area of the cylinder}}$

$= \frac{\pi r\sqrt{r^2 + h^2}}{2\pi r(2r)}$

$= \frac{\sqrt{r^2 + (6r)^2}}{4r}$

$= \frac{\sqrt{37}}{4}$

Thus, the required ratio is $\sqrt{37} : 4$.

7. Let r and h be the base radius and the height of the circular cone respectively.

Then the radius of the hemisphere is r.

$2 \times \frac{2}{3}\pi r^3 = 3 \times \frac{1}{3}\pi r^2 h$

$h = \frac{4}{3}r$

$\frac{\text{total surface area of the hemisphere}}{\text{total surface area of the circular cone}}$

$= \frac{3\pi r^2}{\pi r^2 + \pi r\sqrt{r^2 + h^2}}$

$= \frac{3\pi r^2}{\pi r^2 + \pi r\sqrt{r^2 + \left(\frac{4}{3}r\right)^2}}$

$= \frac{3}{1 + \frac{5}{3}}$

$= \frac{9}{8}$

Thus, the required ratio is 9 : 8.

8. The radius of the upper face of the frustum $B'C'E'F'$

$= 8 \times \frac{24}{32}$

$= 6$ cm

The height of the frustum $B'C'E'F'$

$= 24 \times \frac{6-3}{6}$

$= 12$ cm

The volume of the frustum $B'C'E'F'$

$= \frac{\pi}{3}(12)[6^2 + (6)(3) + 3^2]$

$= 252\pi$

$\approx 792 \text{ cm}^3$

9. Let the side lengths of the cube and the container be $2k$ and $5k$ respectively.
Note that the water level in the container when the cube is just covered by water is $2k$.
The water level in the container when the cube is removed

$$= \frac{(5k)^2(2k) - (2k)^3}{(5k)^2}$$

$$= \frac{42}{25}k$$

The required percentage decrease in water level

$$= \frac{2k - \frac{42}{25}k}{2k} \times 100\%$$

$= 16\%$

10. The height of the vessel

$$= \sqrt{15^2 - 9^2}$$

$= 12$ cm
The curved surface area of the vessel
$= \pi(9)(15)$
$= 135\pi \text{ cm}^2$

Let d cm be the original water depth.

$$\left(\frac{12-d}{12}\right)^2 = \frac{135\pi - [156\pi - \pi(9)^2]}{135\pi}$$

$d = 4$

Let d' cm be the new water depth.

$$\left(\frac{d}{12}\right)^3 = \frac{12^3 - (12-4)^3}{12^3}$$

$d \approx 10.67360659$
Thus, the required water depth is 10.7 cm.

11. (a) $\left(\dfrac{h-7}{h}\right)^3=\dfrac{1}{1+7}$

$h = 14$

(b) Let V cm^3 be the volume of the smaller circular cone.

$\dfrac{V}{V+9408\pi}=\dfrac{1}{8}$

$V = 1\ 344\pi$

The base radius of the smaller circular cone

$=\sqrt{\dfrac{1344\pi}{\frac{1}{3}\pi(14-7)}}$

= 24 cm

The curved surface area of the smaller circular cone

$=\pi(24)\sqrt{24^2+7^2}$

$= 600\pi$ cm

Let A cm^2 be the curved surface area of the frustum.

$\dfrac{A+600\pi}{600\pi}=\left(\dfrac{14}{7}\right)^2$

$A = 1\ 800\pi$

Thus, the required curved surface area is $1\ 800\pi$ cm^2.

12. Note that the part of the pyramid immersed in water is in the form of a frustum.

The side length of the upper face of the frustum

$= 12 \times \dfrac{20-15}{20}$

= 3 cm

The volume of the frustum

$=\dfrac{1}{3}(15)[12^2+(12)(3)+3^2]$

$= 945$ cm^3

The required water level

$= 15-\dfrac{945}{12^2}$

= 8.4375 cm | r.t. 8.44 cm

13. (a) Note that the water surface is in the form of a circle.

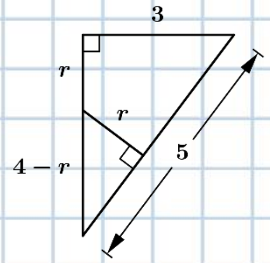
The base radius of the water surface

$=\sqrt{5^2-4^2}$

$= 3$ cm

Let r cm be the radius of the metal sphere.

By considering similar triangles, we have



$\frac{r}{3}=\frac{4-r}{5}$

$r = 1.5$

Thus, the radius of the metal sphere is 1.5 cm.

(b) The volume of the water

$=\frac{1}{3}\pi(3)^2(4)$

$= 12\pi\text{ cm}^3$

The volume of the metal sphere

$=\frac{4}{3}\pi(1.5)^3$

$= 4.5\pi\text{ cm}^3$

Let d cm be the original water level.

$\left(\frac{d}{4}\right)^3=\frac{12\pi-4.5\pi}{12\pi}$

$d=2\sqrt[3]{5}$

$d \approx 3.419951893$

Thus, the required water level is 3.42 cm.

(c) The total volume of water that just covers the metal sphere

$=\frac{1}{3}\pi(12)^2(16)\left[\frac{16^3-(16-1.5\times 2)^3}{16^3}\right]$

$=\frac{5697}{16}\pi\text{ cm}^3$

The required volume of water

$=\frac{5697}{16}\pi-12\pi$

$=\frac{5505}{16}\pi\text{ cm}^3$

14. (a) The radius of the base of the frustum

$= (10)\left(\frac{50-15}{50}\right)$

$= 7$ cm

The capacity of the vessel

$= \frac{\pi}{3}(15)[10^2 + (10)(7) + 7^2]$

$= 1\ 095\pi \text{ cm}^3$

(b) (i) When the water level just covers the hemisphere, the total volume of the water and the hemisphere

$= 1\ 095\pi\left[\frac{(50-15+7)^3-(50-15)^3}{50^3-(50-15)^3}\right]$

$= \frac{31213}{75}\pi \text{ cm}^3$

The required volume of water

$= \frac{31213}{75}\pi - \frac{2}{3}\pi(7)^3$

$= \frac{14063}{75}\pi \text{ cm}^3$

(ii) Let d cm be the required water level.

$$\frac{(d+50-15)^3}{50^3-(50-15)^3} = \frac{\frac{14063}{75}\pi + 1095\pi\left[\frac{(50-15)^3}{50^3-(50-15)^3}\right]}{1095\pi}$$

$d = \sqrt[3]{56938} - 35$

$d \approx 3.471052621$

Thus, the required water level is 3.47 cm.

15. (a) $BD=\sqrt{12^2+16^2}=20$ cm

The height of the pyramid *VABCD*

$$=\sqrt{26^2-\left(\frac{20}{2}\right)^2}$$

= 24 cm

The volume of the pyramid *VABCD*

$$=\frac{1}{3}(12)(16)(24)$$

= 1 536 cm^3

(b) (i) Let V cm^3 be the volume of water

$$\frac{2V}{1536}=\frac{24^3-(24-14)^3}{24^3}$$

$$V=\frac{6412}{9}$$

Let d cm be the required water level.

$$\frac{24^3-(24-d)^3}{24^3}=\frac{\frac{6412}{9}}{1536}$$

$$d=24-\sqrt[3]{7412}$$

$$d\approx 4.502520598$$

Thus, the required water level is 4.50 cm.

(ii) Let r cm be the base radius of the cylinder.

$$\frac{\frac{6412}{9}+\pi r^2(d+0.5)}{1536}=\frac{24^3-(24-d-0.5)^3}{24^3}$$

$$d\approx 1.982170783$$

Thus, the base radius of the cylinder is 1.98 cm.