PERMUTATION, COMBINATION AND PROBABILITY

Form 6
Vol 8

Part 2B - Probability

1.	D

1. The required probability

$$=\frac{3}{8}\times\frac{5}{8}+\frac{5}{8}\times\frac{4}{8}$$

$$=\frac{35}{64}$$

2. (a) The required probability

$$=\frac{C_3^4}{C_3^9}$$

$$=\frac{1}{21}$$

(b) The required probability

$$=\frac{C_2^4\times C_1^5}{C_3^9}$$

$$=\frac{5}{14}$$

(c) The required probability

$$=\frac{4}{9}\times\frac{5}{8}\times\frac{7}{7}+\frac{5}{9}\times\frac{4}{8}\times\frac{7}{7}$$

$$=\frac{5}{9}$$

3. (a) The required probability

$$=\left(\frac{4}{16}\right)^2\times\left(\frac{12}{16}\right)^2\times\frac{4!}{2!\,2!}$$

$$=\frac{27}{128}$$

(b) The required probability

$$=1-\left(\frac{12}{16}\right)^4$$

$$=\frac{175}{256}$$

(c) The required probability

$$=1-\left(\frac{12}{16}\right)^4\times 2+\left(\frac{8}{16}\right)^4$$

$$=\frac{55}{128}$$

(d) The required probability

$$=\left(\frac{12}{16}\right)^4+\frac{4}{16}\times\left(\frac{12}{16}\right)^3\times 4$$

$$=\frac{189}{256}$$

(e) The required probability

$$=\left(\frac{4}{16}\right)^4\times 4!$$

$$=\frac{3}{32}$$

(f) The required probability

$$=C_2^4\times\left(\frac{4}{16}\right)^4\times\left(\frac{4!}{3!}\times 2+\frac{4!}{2!\,2!}\right)$$

$$=\frac{21}{64}$$

4. (a) The required probability

$$= \frac{C_2^4 \times C_1^{12}}{C_3^{16}}$$

$$= \frac{9}{70}$$

(b) The required probability

$$= 1 - \frac{C_3^{12}}{C_3^{16}}$$

$$= \frac{17}{28}$$

(c) The required probability

$$= \frac{C_1^2 \times C_2^4 \times C_1^4 + C_1^4 \times C_1^4 \times C_1^8}{C_3^{16}}$$

or $1 - \frac{C_3^{12}}{C_3^{16}} \times 2 + \frac{C_3^8}{C_3^{16}}$

$$= \frac{11}{35}$$

(d) The required probability

$$= 1 - \frac{C_3^4}{C_3^{16}}$$

$$= \frac{139}{140}$$

(e) The required probability

$$= \frac{C_3^4 \times C_1^4 \times C_1^4 \times C_1^4}{C_3^{16}}$$

$$= \frac{16}{35}$$

(f) The required probability

$$= \frac{P_2^4 \times C_1^4 \times C_2^4}{C_3^{16}}$$

$$= \frac{18}{35}$$

5. (a) The required probability

$$= \frac{C_8^{20} \times 2^8}{C_8^{40}}$$

$$= \frac{512}{1221}$$

(b) The required probability

$$= \frac{C_2^{20} \times C_4^{18} \times 2^4}{C_8^{40}}$$

$$= \frac{640}{5291}$$

(c) The required probability

$$= \frac{512}{1221} + \frac{C_1^{20} \times C_6^{19} \times 2^6}{C_8^{40}} + \frac{640}{5291}$$

$$= \frac{5248}{5291}$$

6. (a) The required probability

$$= \frac{4! \times 6!}{9!}$$

$$= \frac{1}{21}$$

(b) The required probability

$$= 1 - \frac{1}{21}$$

$$= \frac{20}{21}$$

(c) The required probability

$$= \frac{4! \times 5!}{9!}$$

$$= \frac{1}{126}$$

7. (a) The required probability

$$= \frac{5! \times 6!}{11!}$$

$$= \frac{1}{462}$$

(b) The required probability

$$= \frac{6! \times 5! \times 2}{11!}$$

$$= \frac{1}{231}$$

(c) The required probability

$$= \frac{C_5^7 \times 5! \times 6!}{11!}$$

$$= \frac{1}{22}$$

8.	B	9.	B	10.	A	11.	C	12.	C	13.	D

8. The required probability

$$= \frac{C_3^6 \times 3! \times 5!}{8!}$$

$$= \frac{5}{14}$$

9. The required probability

$$= \frac{\left[(8)(1) + (7)(2) + (6)(3) + (5)(4)\right] \times 2}{C_3^{12}}$$

$$= \frac{6}{11}$$

10. The required probability

$$= \frac{4! \times 5!}{9!}$$

$$= \frac{1}{126}$$

11. The required probability

$$= \frac{C_2^{10} C_3^8 + C_1^{10} C_4^8 + C_5^8}{C_5^{18}}$$

$$= \frac{13}{34}$$

12. The required probability

$$= 1 - \frac{C_4^{11} + C_3^{11} \times C_1^4}{C_4^{15}}$$

$$= \frac{25}{91}$$

13. The required probability

$$= C_2^3 \times (\frac{2}{3})^2 \times (1 - \frac{2}{3}) + (\frac{2}{3})^3$$

$$= \frac{20}{27}$$

14. The required probability

$$= (1 - \frac{1}{10}) \times \frac{1}{3} + (1 - \frac{1}{10})^2 \times (1 - \frac{1}{3}) \times (1 - \frac{1}{6}) \times \frac{1}{3} + (1 - \frac{1}{10})^3 \times (1 - \frac{1}{3})^2 \times (1 - \frac{1}{6})^2 \times \frac{1}{3} + \cdots$$

$$= \frac{\frac{9}{10} \times \frac{1}{3}}{1 - \frac{9}{10} \times \frac{2}{3} \times \frac{5}{6}}$$

$$= \frac{3}{5}$$

15. (a) The possible outcomes that the number obtained is a prime number are: 2, 3, 5, 7 and 7.
The required probability

$$= \frac{5}{8}$$

(b) The required probability

$$= (1 - \frac{5}{8}) \times \frac{5}{8} + (1 - \frac{5}{8})^3 \times \frac{5}{8}$$

$$= \frac{1095}{4096}$$

(c) The required probability

$$= (1 - \frac{5}{8}) \times \frac{5}{8} + (1 - \frac{5}{8})^3 \times \frac{5}{8} + (1 - \frac{5}{8})^5 \times \frac{5}{8} + \cdots$$

$$= \frac{\frac{3}{8} \times \frac{5}{8}}{1 - (\frac{3}{8})^2}$$

$$= \frac{3}{11}$$

Part 3 – Conditional Probability

1.	A	2.	A	3.	C

1. The required probability

$$= \frac{C_2^3 \times (\frac{1}{2})^3}{1-(\frac{1}{2})^3}$$

$$= \frac{3}{7}$$

2. The required probability

$$= \frac{\frac{16}{36} \times \frac{15}{35}}{\frac{16}{36}}$$

$$= \frac{3}{7}$$

3. The table below shows the possible outcomes that the sum of the two number thrown is an even number:

	1	2	3	4	5	6
1	2		4		6	
2		4		6		8
3	4		6		8	
4		6		8		10
5	6		8		10	
6		8		10		12

Also, the possible outcomes that at least one number thrown is 4 is highlighted as shown in the above table.

Thus, the required probability is $\frac{5}{18}$.

Part 4 – Expected Value

1.	B	2.	B

1. The possible outcomes of selecting 3 banknotes are: {\$10, \$10, \$10}, {\$10, \$10, \$20}, {\$10, \$20, \$20} and {\$20, \$20, \$20};
   The corresponding amounts of the 3 banknotes are \$30, \$40, \$ \$50 and \$60 respectively; while the corresponding probabilities are $\frac{C_3^7}{C_3^{13}}$, $\frac{C_2^7 \times C_1^6}{C_3^{13}}$, $\frac{C_1^7 \times C_2^6}{C_3^{13}}$ and $\frac{C_3^6}{C_3^{13}}$ respectively.
   The expected amount of the 3 selected banknotes
   $= 30 \times \frac{35}{286} + 40 \times \frac{63}{143} + 50 \times \frac{105}{286} + 60 \times \frac{10}{143}$
   ≈ 43.84615385
   $\approx \$43.8$

2. The probabilities of getting 1 black ball, 2 black balls and 3 black balls are $\frac{C_1^3 \times C_2^2}{C_3^5}$, $\frac{C_2^3 \times C_1^2}{C_3^5}$ and $\frac{C_3^3}{C_3^5}$ respectively.
   The expected number of black balls drawn
   = (1)(0.3)+(2)(0.6)+(3)(0.1)
   = 1.8

3. (a) (i) The required probability
   $= \frac{1}{3} \times \frac{1}{6} \times 2$
   $= \frac{1}{9}$
   (ii) The required probability
   $= \frac{1}{3} \times \frac{5}{6} + \frac{1}{3} \times \frac{1}{6}$
   $= \frac{1}{3}$
   (b) The expected number of tokens got
   $= 180 \times \frac{1}{9} + 30 \times \frac{1}{3}$
   $= 30$