PERMUTATION, COMBINATION AND PROBABILITY

Form 6
Vol 8

Part 1B – Permutation and Combination

1. (a) The required number of arrangements
$= 7!$
$= 5\,040$

(b) The required number of arrangements
$= 5! \times 3 \times 4$
$= 1\,440$

(c) The required number of arrangements
$= 5! \times (C_2^3 + C_2^4) \times 2!$
$= 2\,160$

2. (a) The required number of ways
$= C_4^{20}$
$= 4\,845$

(b) The required number of ways
$= C_4^{10} \times 2^4$
$= 3\,360$

(c) The required number of ways
$= C_1^{10} \times C_2^9 \times 2^2$
$= 1\,440$

(d) The required number of ways
$= C_4^{10} \times (1 + C_3^4 \times C_1^1)$
$= 1\,050$

3. (a) The required number

$= C_4^{20} \times C_3^{15}$

$= 2\ 204\ 475$

(b) The required number

$= C_2^{18} \times C_3^{15}$

$= 69\ 615$

(c) The required number

$= C_4^{20} \times C_3^{13}$

$= 1\ 385\ 670$

(d) Number of different committees can be formed if {Ben and Billy are selected} and {Ada and Ann are not selected} = $C_2^{18} \times C_3^{13} = 43\ 758$

The required number

$= 69\ 615 + 1\ 385\ 670 - 43\ 758$

$= 1\ 411\ 527$

4. (a) The require number of arrangements

$= C_4^4 \times C_2^6 \times 6!$

$= 10\ 800$

(b) The required number of arrangements

$= (P_4^4 \times P_2^6 \times 2) + (P_3^4 \times P_3^6 \times 2) + (P_2^4 \times P_4^6 \times 2) + (P_1^4 \times P_5^6 \times 2) + P_6^6$

$= 22\ 320$

5. (a) The required number

$= C_5^6 \times 5^5$

$= 18\ 750$

(b) The required number

$= C_2^6 \times (C_4^5 \times C_1^5 \times 2 + C_3^5 \times C_2^5 \times 2)$

$= 3\ 750$

(c) The required number

$= C_1^6 \times C_5^5 + 3\ 750 + C_3^6 \times (C_3^5 \times C_1^5 \times C_1^5 \times 3 + C_2^5 \times C_2^5 \times C_1^5 \times 3)$

$= 48\ 756$

6. (a) The required number of arrangements
$= 3^4$
$= 81$

(b) The required number of arrangements
$= 5^3 \times 3 \times 4$
$= 1500$

(c) The required number of arrangements
$= 7^2 \times C_2^3$
$= 147$

Part 2A - Probability

1. B	2. D	3. D

1. The required probability
$= 0.6 \times (1 - 0.6) + (1 - 0.6) \times 0.6$
$= 0.48$

2. The required probability
$= \frac{6}{9} \times \frac{5}{6} + \frac{3}{9} \times \frac{6}{6}$
$= \frac{8}{9}$

3. The required probability
$= 1 - \frac{8}{12} \times \frac{7}{11}$
$= \frac{19}{33}$

4. The required probability
$= \left(\frac{1}{2}\right)^8 \times \frac{8!}{6!\,2!}$
$= \frac{7}{64}$

5. (a) The required probability

$$= \frac{5}{5} \times \frac{6}{6} \times \frac{2}{6}$$

$$= \frac{1}{3}$$

(b) The required probability

$$= \frac{1}{5} \times \frac{6}{6} \times \frac{2}{6}$$

$$= \frac{1}{15}$$

6. (a) The required probability

$$= 0.15^2$$

$$= 0.0225$$

(b) The required probability

$$= (1 - 0.2^2) \times (1 - 0.15^2) \times (1 - 0.4^2)$$

$$= 0.788256 \quad | \text{ r.t. } 0.788$$

(c) The required probability

$$= 0.2^2 \times (1 - 0.15^2) \times (1 - 0.4^2) + (1 - 0.2^2) \times 0.15^2 \times (1 - 0.4^2) + (1 - 0.2^2) \times (1 - 0.15^2) \times 0.4^2$$

$$= 0.201132 \quad | \text{ r.t. } 0.201$$

7. (a) The required probability

$$= \frac{C_2^4}{C_2^9}$$

$$= \frac{1}{6}$$

(b) The required probability

$$= \frac{C_1^4 \times C_1^5}{C_2^9}$$

$$= \frac{5}{9}$$

(c) The required probability

$$= \frac{4}{9} \times \frac{5}{8} + \frac{5}{9} \times \frac{4}{8}$$

$$= \frac{5}{9}$$

(d) The required probability

$$= 1 - \frac{C_2^5}{C_2^9}$$

$$= \frac{13}{18}$$

8. (a) The required probability

$$= 1 - \frac{C_2^6}{C_2^{10}}$$

$$= \frac{2}{3}$$

(b) The required probability

$$= \left(\frac{C_2^4}{C_2^{10}}\right)^2 + \left(\frac{C_1^4 \times C_1^6}{C_2^{10}}\right)^2 + \left(\frac{C_2^6}{C_2^{10}}\right)^2$$

$$= \frac{31}{75}$$

9.	C	10.	C

9. The possible outcomes of the 3 coins are:
{1, 2, 2}, {1, 2, 5}, {1, 2, 10}, {1, 5, 10}, {2, 2, 5}, {2, 2, 10} and {2, 5, 10}.
The corresponding values are:
\$5, \$8, \$13, \$ 16, \$9, \$14, \$17.
The required probability

$$= \frac{4}{7}$$

10. The required probability

$$= (1 - 0.6)^6 + C_1^6 \times 0.6 \times (1 - 0.6)^5 + C_2^6 \times 0.6^2 \times (1\text{-}0.6)^4$$

$$= 0.1792$$