REGULAR QUIZ 04

Form 6

Polynomials, Exponential and Logarithmic Function

MC (@3 marks)

1.	C	Let $f(x)=(2x+1)Q(x)$. $f(x-3)=[2(x-3)+1]Q(x-3)$ $=(2x-5)Q(x-3)$ Thus, $2x-5$ is a factor.
2.	B	Let $p(x)=(x^2+5x+11)(x+c)$. By comparing coefficient of x, we have $5c+11=-4$ $c=-3$ By comparing coefficient of x^2 and the constant term, we have $a=c+5=2$ $b=11c=-33$ $p(x)=x^3=2x^2-4x-33$ Let the required remainder be $mx+n$. $\begin{cases} p(2)=2m+n=-25 \\ p(1)=m+n=-34 \end{cases}$ $m=9$, $n=-43$ The required remainder is $9x-43$.
3.	C	Let $f(x)=(x+3)(x-5)(x+k)$. By comparing coefficient of x^2, we have $3-5+k=2$ $k=4$ $f(x)=(x+3)(x-5)(x+4)$
4.	D	$f(2)=3$ $[f(2)]^3=3^3=27$
5.	B	$\sqrt{a^3\sqrt{b}}\div\sqrt[3]{b^{-1}\sqrt{a^3}}$ $=(a^3b^{\frac{1}{2}})^{\frac{1}{2}}\div(b^{-1}a^{\frac{3}{2}})^{\frac{1}{3}}$ $=a^{\frac{3}{2}}b^{\frac{1}{4}}\div b^{-\frac{1}{3}}a^{\frac{1}{2}}$ $=ab^{\frac{7}{12}}$

6.	C	$54^m = 2^m \times 3^{3m}$ $= 4^{\frac{m}{2}} \times 3^{3m}$ $= \sqrt{x} y^3$
7.	B	II only, the period is reduced to 180°.
8.	D	$\dfrac{4\log_3 x^2 y}{\log_9 \sqrt{xy^3} + \frac{5}{2}\log_9 x}$ $= \dfrac{4\log_3 x^2 y}{\frac{1}{4}\log_3 xy^3 + \frac{5}{4}\log_3 x}$ $= \dfrac{4\log_3 x^2 y}{\frac{1}{4}\log_3 x^6 y^3}$ $= \dfrac{4\log_3 x^2 y}{\frac{3}{4}\log_3 x^2 y}$ $= \dfrac{16}{3}$
9.	B	The equation of L: $0 = \log_a (x - 2a)$ $a^0 = x - 2a$ $x = 2a + 1$ $BC = \log_2 (2a + 1 - 1)$ $5 = \log_2 (2a)$ $2a = 2^5$ $a = 16$
10.	B	$\begin{cases} 60x(1+k\%) + 90y(1-40\%) = 66.6(x+y) \\ 60x(1-k\%) + 90y(1+30\%) = 73.8(x+y) \end{cases}$ $60x(2) + 90y(1.9) = 140.4(x+y)$ $20.4x = 30.6y$ $x : y = 3 : 2$
11.	A	$\left(\dfrac{OA}{OB}\right)^2 = \dfrac{5\pi}{9.8\pi}$ $OA : OB = 5 : 7$ The required perimeter $= \dfrac{5\pi}{5} \times 2 + \dfrac{9.8\pi}{7} \times 2 + 12 \times 2 = 4.8\pi + 24 \text{ cm}$

12.	A	Let O be the centre of the circle. $OA=\sqrt{OC^2-AC^2}=\sqrt{15^2-12^2}=9\text{ cm}$ $OB=9+3=12\text{ cm}$ $BD=\sqrt{OD^2-OB^2}=\sqrt{15^2-9^2}=12\text{ cm}$ $\angle COA=\tan^{-1}\frac{12}{9}$ $\angle DOB=\tan^{-1}\frac{9}{12}$ The area of the shaded region: $\pi\frac{\angle COA}{360^\circ}\times 15^2-\frac{OA\times AC}{2}-\left(\pi\frac{\angle DOB}{360^\circ}\times 15^2-\frac{OB\times BD}{2}\right)$ $=31.9\text{ cm}^2$
13.	D	Lower limit $=65\times(1-0.4\%)=64.74\text{ g}$ Upper limit $=65\times(1+0.4\%)=65.26\text{ g}$
14.	B	By mid-pt thm., we have $PQ=\frac{1}{2}AB$ and $QR=\frac{1}{2}CD$ Thus, $PQ=QR$. Also, $PQ\,//\,AB$ and $QR\,//\,CD$ $\angle PQC=70^\circ$ and $\angle RQC=80^\circ$ Thus, $\angle PRQ=15^\circ$
15.	C	Choice C may not be true unless $ABCD$ is a rhombus.
16.	D	When $a>0$ and $x_1>x_2$, then $0<r<1$. When $a<0$ and $x_1>x_2$, then $r>1$. Thus, I may not true. When $0<r<1$, then $y_2<y_1$. Thus, II may not true. When $r>1$, then $z_1<z_2$ Thus, III may not true.

1.	C	2.	B	3.	C	4.	D	5.	B
6.	C	7.	B	8.	D	9.	B	10.	B
11.	A	12.	A	13.	D	14.	B	15.	C
16.	D								

Long Questions (32 marks)

17. (a) $(a-b+11)^2$ 1A

(b) $8^2-(2-4x)^2$

$=(8+2-4x)(8-2+4x)$ 1M

$=(10-4x)(6+4x)$

$=4(5-2x)(3+2x)$ 1M+1A

(4)

18. (a) $2^{2x}+2\times 2^{2x}=24$ 1M

$3\times 2^{2x}=24$

$2^{2x}=8$

$2x=3$ 1M

$x=\dfrac{3}{2}$ 1A

(b) $\log_3 2x-\dfrac{\log_3 6x^2}{\log_3 9}=\dfrac{\log_3 \frac{x}{3}}{\log_3 27}+1$ 1M

$\log_3 2x-\log_3 (6x^2)^{\frac{1}{2}}-\log_3 \left(\dfrac{x}{3}\right)^{\frac{1}{3}}=1$ 1M

$\log_3 \dfrac{2x}{(6x^2)^{\frac{1}{2}}\left(\frac{x}{3}\right)^{\frac{1}{3}}}=1$ 1M

$\dfrac{2\times 3^{\frac{1}{3}}}{6^{\frac{1}{2}}x^{\frac{1}{3}}}=3$ 1M

$x^{\frac{1}{3}}=\dfrac{2\times 3^{\frac{1}{3}}}{6^{\frac{1}{2}}\times 3}$

$x=\dfrac{2^{\frac{3}{2}}}{3^{\frac{7}{2}}}$ 1A

(8)

19. (a) $\log_3 y = n\log_3 x + \log_3 k$ 1M

$\log_3 y = 2n\log_9 x + \log_3 k$ 1M

$\log_3 k = 4$

$k = 3^4 = 81$ 1A

$2n = \frac{4}{3}$

$n = \frac{2}{3}$ 1A

(b) Total Cost $= 81(8000)^{\frac{2}{3}}$ 1M

$= \$32400$ 1A

(c) Total Cost $= 81(350)^{\frac{2}{3}}$

≈ 4023

> 4000

$\therefore$ No, it is not possible. 1A f.t.

(7)

20. (a) $S(6) = 28 \times S(3)$

$\frac{a(r^6 - 1)}{r - 1} = \frac{28a(r^3 - 1)}{r - 1}$ 2M

$r^6 - 1 = 28r^3 - 28$

Solving, we have $r^3 = 27$ or $r^3 = 1$ 1M

$r = 3$ or $r = 1(\text{rej.})$ 1A

(b) $a = \frac{1}{3}$

$T(6) + \ldots + T(10)$

$= S(10) - S(5)$

$= \frac{\frac{1}{3}(3^{10} - 1)}{3 - 1} - \frac{\frac{1}{3}(3^5 - 1)}{3 - 1}$ 2M

$= 9801$ 1A

(7)

21. (a) $p(x)=(x^2+px+q)(x-2)(x+4)+r$

$p(2)=p(-4)=r$

$(2+3)(2^3+2^2m+2n-1)=(-4+3)((-4)^3+(-4)^2m-4n-1)$ 1M

$40+20m+10n-5=64-16m+4n+1$

$36m+6n=30$

$6m+n=5\ldots(1)$

$p(1)=p(-1)$

$(1+3)(1+m+n-1)=(-1+3)(-1+m-n-1)$ 1M

$4+4m+4n-4=-2+2m-2n-2$

$2m+6n=-4$

$m+3n=-2\ldots(2)$

Solving

$m=1$, $n=-1$ 1A+1A

(b) $p(x)=(x+3)(x^3+x^2-x-1)$

$=(x+3)(x^2-1)(x+1)$ 1M

$=(x+1)^2(x-1)(x+3)$ 1A

(6)