TRANSFORMATION OF GRAPH

Form 5

Vol 8

Part 3B – Enlargement / Reduction

1.	D	2.	C	3.	D	4.	D

1. Let $f(x) = x^2 + 8x + 6$.

Then, we have $f(x) = (x + 4)^2 - 10$.

$y = 4x^2 + 8x - 6$

$= 4(x + 1)^2 - 10$

$= (2x + 2)^2 - 10$

$= [2(x - 1) + 4]^2 - 10$

Thus, the graph of $y = 4x^2 + 8x - 6$ is obtained by reducing the graph of $y = f(x)$ to $\frac{1}{2}$ of its original along the x-axis and then translating the resulting graph rightward by 1 unit.

2. Let $f(x) = 4x^2 - 20x + 12$.

Then, we have $f(x) = 4(x - \frac{5}{2})^2 - 13$ and vertex of $f(x)$ is $\left(\frac{5}{2}, -13\right)$

Let $g(x) = x^2 + 4x - 9$.

Then, we have $g(x) = (x + 2)^2 - 13$ and vertex of $g(x)$ is $(-2, -13)$

Since the coefficient of x^2 term of $f(x)$ and $g(x)$ are 4 and 1 respectively, so the graph of $f(x)$ is enlarged 2 times along the x-axis.

The required transformation is either

(1) translate $\frac{7}{2}$ units to the left $\left(\frac{5}{2}, -13\right) \to (-1, -13)$, and then enlarge to 2 times along the x-axis $(-1, -13) \to (-2, -13)$, or

(2) enlarge to 2 times along the x-axis $\left(\frac{5}{2}, -13\right) \to (5, -13)$, and then translate 7 units to the left $(5, -13) \to (-2, -13)$.

Thus, the answer is C.

3. Let $f(x) = (x - a)^2 - 4a + a^2$ and $g(x) = 4x^2 - 8ax + 4a^2$.

Then $g(x) = k\,f(x) + 12$, where k is a constant.

Since the coefficient of x^2 term in $g(x)$ is 4, we have $k = 4$.

Note that $g(x) = 4(x - a)^2$.

So, we have $4(-4a + a^2) + 12 = 0$.

By solving, we have $a = 1$ or $a = 3$.

4. Let $f(x) = 9(x^2 + (2a + 4)x + a^2 + 4a - 5)$ and $g(x) = x^2 - 16x + 7 - 6a^2$.

Then $g(x) = f[h(x - k)]$, where h is a constant.

Note that $f(x) = (3x)^2 + (6a + 12)(3x) + 9a^2 + 36a - 45 = (3x + 3a + 6)^2 - 81$ and

$g(x) = (x - 8)^2 - 57 - 6a^2$.

So, we have $-81 = -57 - 6a^2$.

Hence, we have $a = -2$ or $a = 2$.

By comparing the terms of degree 2, we have $h = \frac{1}{3}$.

For $a = -2$, we have $-k + 3(-2) + 6 = -8$.

$\therefore\ k = 8$

For $a = 2$, we have $-k + 3(2) + 6 = -8$.

$\therefore\ k = 20$

Thus, $k = 8$ or $k = 20$.

Part 4 - Trigonometric Graph

1. A	2. C	3. C	4. B	5. D	6. D	7. A
8. C	9. B	10. D	11. D	12. D	13. C	14. A
15. B	16. D	17. C	18. D	19. B		

1. Note that the graph cuts the x-axis at $x = 0°$, $180°$ and $360°$.
So, the required function is in the form of $y = k \sin x$, where k is a constant.
Since $k \sin 90° = -2$, we have $k = -2$.
Thus, the required function is $y = -2 \sin x$.

2. The graph of $y = -f(2x)$ is obtained by reflecting the graph of $y = f(x)$ along the x-axis and reducing the graph to $\frac{1}{2}$ of its original along the x-axis.
Hence, the graph of $y = -f(2x)$ must cut the negative y-axis and be compressed along the x-axis.
Thus, C is the desired answer.

3. We can observe that the graph of $y = q(x)$ is obtained by reflecting the graph of $p(x)$ along the x-axis, followed by reducing to $\frac{1}{2}$ of its original along the x-axis and reducing to $\frac{1}{2}$ of its original along the y-axis.
Thus, we have $q(x) = -\frac{1}{2}p(2x)$.

4. We can observe that the graph is obtained by reflecting the graph of $y = \sin x$ along the x-axis and then translating the resulting graph upwards by k units.
Hence, the graph represents the function in the form of $y = -\sin x + k$.
Since $-\sin 360° + k = 2$, we have $k = 2$.
Thus, the figure shows the graph of $y = 2 - \sin x$.

5. By substituting $x = 290°$ and observe that
A: $\sin(290° + 20°) = \sin 310° \neq 1$
B: $\sin(290° - 20°) = \sin 270° = -1 \neq 1$
C: $\cos(290° - 20°) = \cos 270° = 0 \neq 1$
D: $\sin(20° - 290°) = \sin(-270°) = 1$
Thus, D is the desired answer.

6. Note that the required function attains its maximum at 300° and attains its minimum at $0° < x < 300°$.
By substituting $x = 300°$ and observe that
A: $-\sin(300° + 30°) = -\sin 330° < 1$
B: $\sin(\frac{300°}{2} - 60°) = \sin 90° = 1$, however $-1 < -\frac{\sqrt{3}}{2} < \sin(\frac{x}{2} - 60°) < 1$.
C: $\cos(300° - 30°) = \cos 270° = 0 < 1$
D: $\cos(300° + 60°) = \cos 360° = 1$ and $\cos(120° + 60°) = \cos 180° = -1$, where $0° < 120° < 360°$.
Thus, D is the desired answer.

7. Let the required function be $y = k \sin(x° + a°) - h$, where a, h and k are constants.
Note that the minimum value of the function is -4.
So, we have $h = 0 - (\frac{-4}{2})$ and $k = \frac{4}{2}$.
Hence, we have $h = 2$ and $k = 2$.
Since $2 \sin(40° + a°) - 2 = 0$, we have $a = 50$ (for $0° < a° < 90°$).
Thus, the figure shows the graph of $y = 2 \sin(x° + 50°) - 2$.

8. We can observe that the graph is obtained by compressing a sine function or a cosine function.
Thus, the options A and B can be eliminated.
By substituting $x = 0°$ and observe that
C: $\sin[2(0°) - 40°] = \sin(-40°) < 0$
D: $\cos[2(0°) + 50°] = \cos 50° > 0$
Thus, C is the desired answer.

9. We can observe that the graph is obtained by stretching a sine function or a cosine function.
Thus, the options A and C can be eliminated.
By substituting $x = 0°$ and observe that
B: $2 \cos[\frac{2}{3}(0°) + 20°] = 2 \cos 20° > 0$
D: $-\sin[\frac{(0°)}{3} + 10°] = -\sin 10° < 0$
Thus, B is the desired answer.

10. We can observe that the graph is obtained by stretching a sine function or a cosine function.
Thus, the options A and B can be eliminated.
By substituting $x = 0°$ and observe that

C: $2\cos[\frac{(0°)}{2} - 30°] + 3 = 2\cos(-30°) + 3 = \sqrt{3} + 3 > 0$

D: $-2\cos[\frac{(0°)}{2} - 30°] + 1 = -2\cos(-30°) + 1 = -\sqrt{3} + 1 < 0$

Thus, D is the desired answer.

11. We can observe that the graph is obtained by stretching a sine function or a cosine function.
Thus, the options A and C can be eliminated.
By substituting $x = 0°$ and observe that

B: $\frac{3}{2}\sin[\frac{(0°)}{2}] + \frac{1}{2} = \frac{1}{2} \neq 0$

D: $3\sin[\frac{(0°)}{2}] - 1 = -1$

Thus, D is the desired answer.

12. Note that $-1 \leq \cos(ax) \leq 1$ and $a > 1$.
So, we have $\cos[a(180°)] + 1 = 2$ and hence $a(180°) = 360°$.
Thus, we have $a = 2$.

13. Note that $-1 \leq \sin(x - 45°) \leq 1$ for $0° \leq x \leq 360°$.
So, we have $-k - 1 \leq k\sin(x - 45°) - 1 \leq k - 1$ for $0° \leq x \leq 360°$.
Since $k - 1 = 1$, we have $k = 2$.

14. Note that $h\sin 90° + k = 5$ and $h\sin 270° + k = -2$.
So, we have $h + k = 5$ and $-h + k = -2$.
Solving, we have $h = \frac{7}{2}$ and $k = \frac{3}{2}$.

15. Note that $k\cos 0° + h = 0$ and $k\cos 180° + h = 4$.
So, we have $k + h = 0$ and $-k + h = 4$.
Solving, we have $h = 2$ or $k = -2$.

16. For $-90° < \theta < 90°$, $\cos(0° + \theta) = \frac{1}{2}$ implies $\theta = -60°$ or $60°$.

Observe that the graph is obtained by translating the graph of $y = \cos x$ to the left.
Thus, we have $\theta = 60°$.

17. Note that $-1 \leq \sin(kx - 30°) \leq 1$.

So, we have $-h + \frac{1}{2} \leq h\sin(kx - 30°) \leq h + \frac{1}{2}$.

Since $h + \frac{1}{2} = 1$, we have $h = \frac{1}{2}$.

Hence, we have $\frac{1}{2}\sin[k(280°) - 30°] + \frac{1}{2} = 1$.

Also observe that the graph completes 3 cycles in $0° < x < 360°$.
Therefore, $\sin[k(280°) - 30°] = 1$ implies $k(280°) - 30° = 90° + 360° \times 2$.
Thus, we have $k = 3$.

18. For $-90° < q < 90°$, we can observe that the graph of $y = \sin(mx + q)$ is obtained by stretching the graph of $y = \sin x$ and then translating the resulting graph to the left.
So, we have $0 < m < 1$ and $q > 0$.
Thus, D is the desired answer.

19. For $-180° < q < 180°$, we can observe that the graph of $y = \cos[m(x + q)]$ is obtained by compressing the graph of $y = \cos x$ and then translating the resulting graph to the right.
So, we have $m > 1$ and $-180° < q < 0°$.
Also observe that the graph completes 2 cycles in $0° < x < 360°$.
Hence, we have $m = 2$.
Since $\cos[2(220° + q)] = 1$, we have $2(220° + q) = 0° + 360°$.
Thus, we have $q = -40°$.