TRANSFORMATION OF GRAPH

Form 5

Vol 8

Part 2B – Reflection

1. (a) $(-4k+4)^2-4(2)(3k^2-4k+3)$

$=-8\left[-2k^2+4k-2+3k^2-4k+3\right]$

$=-8(k^2+1)$

Since $-8(k^2+1)<0$ for all real k, the equation of $f(x)=0$ does not have real roots and the graph of $y=f(x)$ does not cut the x-axis.

(b) $f(x)=2\left[x^2-2(k-1)x+(k-1)^2-(k-1)^2\right]+3k^2-4k+3$

$=2(x-k+1)^2+k^2+1$

Thus, the coordinates of the required vertex are $(k-1,k^2+1)$.

(c) Note that when A and B are nearest to each other, they coincide with vertices of the graphs of $y=f(x)$ and $y=g(x)$ respectively.

The coordinates of A are $(-1+k,1+k^2)$ and that of B are $(-1+k,1-k^2)$.

Note that BT is horizontal and the x-coordinate of the mid-point of BT is 0.

The perpendicular bisector of BT is $x=0$.

Hence, the circumcentre of ΔATB must lie on the line $x=0$.

Thus, the claim is not correct.

2. (a) $f(-6)$

$=\dfrac{1}{1-k}\left((-6)^2+(16-4k)(-6)+(22+14k)\right)$

$=\dfrac{1}{1-k}(-38+38k)$

$=-38$

Thus, the graph of $y=f(x)$ passes through S.

(b) (i) $g(x)$

$=f(-x)+26$

$=\frac{1}{1-k}\left((-x)^2+(16-4k)(-x)+(22+14k)\right)+26$

$=\frac{1}{1-k}\left(x^2-(16-4k)x+(8-2k)^2-(8-2k)^2+(22+14k)\right)+26$

$=\frac{1}{1-k}\left((x-8+2k)^2+(1-k)(4k-42)\right)+26$

$=\frac{1}{1-k}(x-8+2k)^2+(4k-16)$

Thus, the coordinates of U are $(8-2k, 4k-16)$.

(ii) The coordinates of T are $(6,-12)$.

Note that the area of the circle passing through S and T is the least when ST is a diameter of the circle.

If U lies on this circle, then we have $\angle SUT=90°$.

Under this case, we have $k\neq 1$ and $k\neq 7$.

$\left(\frac{(4k-16)+12}{(8-2k)-6}\right)\left(\frac{(4k-16)+38}{(8-2k)+6}\right)=-1$

$\frac{(4k-4)(4k+22)}{(2-2k)(14-2k)}=-1$

$(k-1)(5k+15)=0$

$k=1$ (rejected) or $k=-3$

Thus, the area of the circle passing through S, T and U is the least when $k=-3$.

(iii) Putting $y=0$ in $5x+4y+100=0$, we have $x=-20$.

So, the coordinates of A are $(-20,0)$.

The product of the slope of AT and the slope of ST

$=\left(\frac{-12-0}{6+20}\right)\left(\frac{-12+38}{6+6}\right)$

$=-1$

So, we have $\angle ATS=90°$.

Since K lies on ST, we have $\angle ATK=90°$.

Note that the x-coordinate of K is 0.

Then, we have K lies on the y-axis and hence $\angle AOK=90°$.

It follows that $\angle ATK=\angle AOK$.

Thus, A, O, T and K are concyclic. (converse of $\angle$s in the same segment)

The claim is agreed.

Part 3A – Enlargement / Reduction

1. A	2. C	3. D	4. C	5. D	6. C	7. C
8. D	9. B	10. C				

1. The graph of $y = (\frac{x}{3})$ is obtained by enlarging the graph of $y = f(x)$ by 3 times along the x-axis.

   So, the coordinates of the vertex of the graph of $y = (\frac{x}{3})$ are (– 3, 2).

   Thus, A is the desired answer.

2. Note that $g(x) = -\frac{1}{2}f(x)$.

   So, the graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ along the x-axis and reducing to $\frac{1}{2}$ of its original along the y-axis.

   Hence, the y-intercept of the graph $y = g(x)$ is $\frac{3}{2}$ while both of its x-intercepts remain unchanged.

   Thus, C is the desired answer.

3. The graph of $y = f(-3x)$ is obtained by reducing the graph of $y = f(x)$ to $\frac{1}{3}$ of its original along the x-axis and reflecting the graph along the y-axis.
   Thus, the x-intercepts of the graph $y = f(-3x)$ are – 1 and 2.

4. Let $f(x) = 4x^2 + 8$.
   The required function is

   $y = \frac{1}{4}f(x)$

   $y = x^2 + 2$

5. Let $f(x) = 2x^2 - x + 4$.
   The required function is

   $y = f(\frac{1}{2}x)$

   $y = 2(\frac{1}{2}x)^2 - (\frac{1}{2}x) + 4$

   $y = \frac{1}{2}x^2 - \frac{1}{2}x + 4$

6. Let $f(x) = 8x^2 + 12x - 9$.

The required function is

$y = f[\frac{1}{2}(x - 10)]$

$y = 8[\frac{1}{2}(x - 10)]^2 + 12[\frac{1}{2}(x - 10)] - 9$

$y = 2x^2 - 34x + 131$

7. $g(x) = 8 \log_7 x$

$\quad = 8f(x)$

Thus, the graph of $y = f(x)$ is vertically stretched by a factor of 8 into the graph of $y = g(x)$.

8. Let $f(x) = \log_2(4x)$. Then, we have

$y = \log_2 x + 1$

$\quad = \log_2(2x)$

$\quad = \log_2[\frac{1}{4}(8x)]$

$\quad = \log_2[\frac{1}{4}(4x)] + \log_2 2$

$\quad = \log_2[\frac{1}{4}(4x)] + 1$

which shows that the graph of $y = \log_2 x + 1$ can be obtained by translating the graph of $y = f(x)$ upward by 1 unit and then enlarging the resulting graph by 4 times along the x-axis.

$y = \log_2 x + 1$

$\quad = \log_2 x + 2 - 1$

$\quad = \log_2(4x) - 1$,

which shows that the graph of $y = \log_2 x + 1$ can be obtained by translating the graph of $y = f(x)$ downward by 1 unit.

$y = \log_2 x + 1$

$\quad = \log_2(2x)$

$\quad = \log_2[\frac{1}{2}(4x)]$,

which shows that the graph of $y = \log_2 x + 1$ can be obtained by enlarging the graph of $y = f(x)$ by 2 times along the x-axis.

Hence, all of the statements I, II and III can be the transformation in the graph of $y = \log_2(4x)$.

9. Let $f(x) = 4\log(2x)$. Then, we have

$y = \log(2x) + 1$

$= \log(20x)$

$= \frac{1}{4} \times 4\log[10(2x)]$

Thus, the graph of $y = \log(2x) + 1$ is obtained by reducing the graph of $y = f(x)$ to $\frac{1}{4}$ of its original along the y-axis and then reducing the resulting graph to $\frac{1}{10}$ of its original along the x-axis.

10. Let $f(x) = 3 \times 8^x$. Then, we have

$y = 12 \times 4^x$

$= 4 \times 3 \times 8^{\frac{2}{3}x}$

$= 8^{\frac{2}{3}} \times 3 \times 8^{\frac{2}{3}x}$

$= 3 \times 8^{\frac{2}{3}(x+1)}$

Thus, the graph of $y = 12 \times 4^x$ is obtained by enlarging the graph of $y = f(x)$ to $\frac{3}{2}$ times of its original along the x-axis and then translating the resulting graph leftward by 1 unit.