POLYNOMIALS, EXPONENTIAL AND LOGARITHMIC FUNCTION

Form 6
Vol 7

Part 10B – Exp/log graphs

1.	D	2.	C	3.	A	4.	C	5.	C	6.	A	7.	B
8.	C	9.	A	10.	A	11.	C	12.	A				

1. $\log_2 y = -\frac{1}{2}x + 5$

$\log_2 y - \log_2 32 = -\frac{1}{2}x$

$\log_2 \frac{y}{32} = -\frac{1}{2}x$

$\frac{y}{32} = 2^{-\frac{x}{2}}$

$y = 32 \cdot 2^{-\frac{x}{2}}$

2. $\log_9 y = \log_3 x + 4$

$\log_9 y = \log_9 x^2 + 4$

$\log_9 \frac{y}{x^2} = 4$

$\frac{y}{x^2} = 6561$

$y = 6561x^2$

3. $y = ax^b$

$\log_a y = \log_a a + \log_a x^b$

$\frac{\log_8 y}{\log_8 a} = 1 + b\log_a x$

$\log_8 y = (b\log_8 a)\log_a x + \log_8 a$

y-intercept:

$\log_8 a = 3$

$a = 512$

slope:

$b\log_8 a = -\frac{3}{2}$

$b = -\frac{1}{2}$

$ab = (512)(-\frac{1}{2}) = -256$

4. $\log_{\frac{1}{4}} y = \frac{1}{2}\log_2 x + 4$

$\log_2 y^{-\frac{1}{2}} = \frac{1}{2}\log_2 x + 4$

$-\frac{1}{2}\log_2 y - \frac{1}{2}\log_2 x = 4$

$\log_2(xy) = -8$

$xy = \frac{1}{256}$

$256xy = 1$

5. $\log_2 y = (\frac{8-4}{0-3})\log_4 x + 8$

$\log_2 y = -\frac{4}{3}\log_2 x^{\frac{1}{2}} + 8$

$\log_2 y + \log_2 x^{\frac{2}{3}} = 8$

$\log_2(yx^{\frac{2}{3}}) = 8$

$yx^{\frac{2}{3}} = 256$

6. According to $y - y_1 = m(x - x_1)$,

$y - 10 - 0 = (\frac{12-0}{12-6})(\frac{1}{x} + x - 6)$

Sub y = -7:

$-7 - 10 = \frac{2}{x} + 2x - 12$

$\frac{2}{x} + 2x + 5 = 0$

$2x^2 + 5x + 2 = 0$

$x = -\frac{1}{2}$ or $x = -2$

7. $y = Ab^x$

$\log_2 y = \log_2 A + x\log_2 b$

From the figure, $0 < A < 1$ and $b > 1$.

slope: $\log_2 b > 0$

y-intercept: $\log_2 A < 0$

8. $y = Ab^x$

$\log_{0.2} y = \log_{0.2} A + x\log_{0.2} b$

From the figure, $0 < A < 1$ and $0 < b < 1$.

slope: $\log_{0.2} b > 0$

y-intercept: $\log_{0.2} A > 0$

9. $y = a^{-1}b^{x+2}$

Sub (0, 0.4):

$0.4 = a^{-1}b^{0+2}$

$a^{-1}b^2 = 0.4$

The required graph:

$y = a^{-1}b^{x+2}$

$\log_{0.5} y = -\log_{0.5} a + (x+2)\log_{0.5} b$

$\log_{0.5} y = x\log_{0.5} b - \log_{0.5} a + 2\log_{0.5} b$

slope: $\log_{0.5} b < 0$

y-intercept: $-\log_{0.5} a + 2\log_{0.5} b = \log_{0.5} a^{-1}b^2 = \log_{0.5} 0.4 > 0$

10. $y = ab^x$

$\log_{\frac{1}{8}} y = \log_{\frac{1}{8}} a + b\log_{\frac{1}{8}} x$

From the figure, the y-intercept $\log_{\frac{1}{8}} a$ is negative and the slope b is positive.

Thus, $a > 1$ and $b > 0$.

The required graph:

$y = ab^x$

$\log_4 y = \log_4 a + b\log_4 x$

slope: $b > 0$

y-intercept: $\log_4 a > 0$

11. $y = mn^x$

$\log_a y = \log_a m + x\log_a n$

slope: $\log_a n = \dfrac{-8-0}{0-2} = 4$

y-intercept: $\log_a m = -8$

I is true:

$\because \log_a m < 0$ and $a > 1$

$\therefore 0 < m < 1$

II is not true:

$\because \log_a n > 0$ and $a > 1$

$\therefore n > 1$

III is true:

$\log_a n = 4$ and $\log_a m = -8$

$n = a^4$ and $m = a^{-8}$

$mn^2 = (a^{-8})(a^4)^2 = 1$

12. $y = mn^{x-1}$

$\log_a y = \log_a m + (x-1)\log_a n$

$\log_a y = x\log_a n + \log_a m - \log_a n$

slope: $\log_a n = \dfrac{3-0}{0-(-9)} = \dfrac{1}{3}$

y-intercept: $\log_a m - \log_a n = 3$

$\log_a m - \dfrac{1}{3} = 3$

$\log_a m = \dfrac{10}{3}$

I is true:

$\because \log_a m > 0$ and $0 < a < 1$

$\therefore 0 < m < 1$

II is not true:

$\because \log_a n > 0$ and $0 < a < 1$

$\therefore 0 < n < 1$

III is not true:

$\log_a n = \dfrac{1}{3}$ and $\log_a m = \dfrac{10}{3}$

$n = a^{\frac{1}{3}}$ and $m = a^{\frac{10}{3}}$

$m^3n^3 = (a^{\frac{1}{3}})^3(a^{\frac{10}{3}})^3 = a^{11}$