POLYNOMIALS, EXPONENTIAL AND LOGARITHMIC FUNCTION

Form 6

Vol 7

Part 9B – Exp/log equations

1. A	2. A	3. D	4. D	5. C	6. C	7. B
8. D	9. B	10. C	11. D	12. D		

1. $\log_5 \frac{x}{25} = -3\log_5 y$

$\log_5 \frac{x}{25} = \log_5 y^{-3}$

$\frac{x}{25} = y^{-3}$

$x = 25y^{-3}$

2. $\log(5x+13) = \log(3x+2)+1$

$\log(5x+13) = \log(3x+2)+\log 10$

$\log(5x+13) = \log 10(3x+2)$

$5x+13 = 30x+20$

$25x = -7$

$x = -\frac{7}{25}$

3. $\log(x+4) = \log 3 + \log(x-2)$

$\log(x+4) = \log 3(x-2)$

$x+4 = 3x-6$

$2x = 10$

$x = 5$

4. $[\log_2(x-4)]^2 + \log_2(x-4)^2 - 3 = 0$

$[\log_2(x-4)]^2 + 2\log_2(x-4) - 3 = 0$

$\log_2(x-4) = 1$ or $\log_2(x-4) = -3$

$x-4 = 2$ or $x-4 = 2^{-3}$

$x = 6$ or $x = \frac{33}{8}$

5. $2+\log_2(2x)=\log_2(3x+10)$

$\log_2 4+\log_2(2x)=\log_2(3x+10)$

$\log_2 4(2x)=\log_2(3x+10)$

$8x=3x+10$

$x=2$

6. $\log_5(8x-11)=2+\log_{\frac{1}{5}}(2x+1)$

$\log_5(8x-11)=2+\log_5(2x+1)^{-1}$

$\log_5(8x-11)=\log_5 25-\log_5(2x+1)$

$8x-11=\dfrac{25}{2x+1}$

$16x^2-14x-11=25$

$16x^2-14x-36=0$

$x=2$ or $x=-\dfrac{9}{8}$ (rej.)

7. $\log_9(2x)=\log_3 x+1$

$\log_9(2x)=\log_9 x^2+\log_9 9$

$2x=9x^2$

$9x^2-2x=0$

$x=0$ (rej.) or $x=\dfrac{2}{9}$

8. $\log_4\dfrac{3x}{2}-\log_8 8x^2=-\dfrac{3}{2}$

$\log_4\dfrac{3x}{2}-\log_4\left(8x^2\right)^{\frac{2}{3}}=-\dfrac{3}{2}$

$\log_4\dfrac{3x}{2}-\log_4 4x^{\frac{4}{3}}=-\dfrac{3}{2}$

$\dfrac{\frac{3x}{2}}{4x^{\frac{4}{3}}}=4^{-\frac{3}{2}}$

$x^{-\frac{1}{3}}=\dfrac{1}{3}$

$x=27$

9. $\dfrac{9}{\log x^3+5}-19=\dfrac{6}{\log x^2-3}$

$9(2\log x-3)-19(3\log x+5)(2\log x-3)=6(3\log x+5)$

$18\log x-27-114(\log x)^2-19\log x+285=18\log x+30$

$114(\log x)^2+19(\log x)-228=0$

$\log x=\dfrac{4}{3}$ or $\log x=-\dfrac{3}{2}$

$-\log x=-\dfrac{4}{3}$ or $-\log x=\dfrac{3}{2}$

$\log\dfrac{1}{x}=-\dfrac{4}{3}$ or $\log\dfrac{1}{x}=\dfrac{3}{2}$

10. $\begin{cases}\log y=\log(x+5)-2\\ \log y^2=\log(2x)-3\end{cases}$

$\log y^2=\log(2x)-3$

$2\log y=\log(2x)-3$

$2[\log(x+5)-2]=\log(2x)-3$

$2\log(x+5)-4=\log(2x)-3$

$\log(x+5)^2-\log(2x)=\log 10$

$\dfrac{(x+5)^2}{2x}=10$

$x^2+10x+25=20x$

$x^2-10x+25=0$

$x=5$

$y=\dfrac{1}{10}$

11. $\begin{cases}x+\log y=2\\ x^2+\log y^2-5=2\end{cases}$

$\begin{cases}\log y=2-x\\ x^2+2\log y-5=2\end{cases}$

$x^2+2\log y-5=2$

$x^2+2(2-x)-5=2$

$x^2-2x-3=0$

$x=3$ or $x=-1$

$y=\dfrac{1}{10}$ or $y=1000$

12. $\begin{cases} 3\log_8 y = x-1 \\ \frac{1}{2}(\log_4 y)^2 = x+5 \end{cases}$

$\begin{cases} 3\log_4 y^{\frac{2}{3}} = x-1 \\ (\log_4 y)^2 = 2x+10 \end{cases}$

$\begin{cases} 2\log_4 y = x-1 \\ (\log_4 y)^2 = 2x+10 \end{cases}$

$(\log_4 y)^2 = 2x+10$

$(\frac{x-1}{2})^2 = 2x+10$

$x^2 - 2x + 1 = 8x + 40$

$x^2 - 10x - 39 = 0$

$x = 13$ or $x = -3$

$y = 4096$ or $y = \frac{1}{16}$

Part 10A – Exp/log graphs

1. C	2. D	3. C				

1. Let Q(k,0).

$R(k, \log_b k)$

$P(k, \log_a k)$

$\frac{PQ}{QR} = \frac{7}{3}$

$\frac{0 - \log_a k}{\log_b k - 0} = \frac{7}{3}$

$\frac{-\log_a k}{\log_b k} = \frac{7}{3}$

$\log_a b = -\frac{7}{3}$

$b = a^{-\frac{7}{3}}$

$b^3 = a^{-7}$

$a^7 b^3 = 1$

2. I is true:

Add line y=1.

The x-coordinates of the intersecting point with $y=\log_a x$ and $y=\log_b x$ are a and b respectively.

From the graph, a < b.

II is true:

Since $y=\log_b x$ is a decreasing function, b < 1.

III is true:

Let A(k,0).

$B(k,\log_a k)$

$C(k,\log_b k)$

$$\frac{AB}{BC}=\frac{0-\log_a k}{\log_a k-\log_b k}$$

$$\frac{AB}{BC}=\frac{\frac{-\log_a k}{\log_a k}}{\frac{\log_a k}{\log_a k}-\frac{\log_b k}{\log_a k}}$$

$$\frac{AB}{BC}=\frac{-1}{1-\log_b a}$$

$$\frac{AB}{BC}=\frac{1}{\log_b a-\log_b b}$$

$$\frac{AB}{BC}=\frac{1}{\log_b \frac{a}{b}}$$

$$\frac{AB}{BC}=\log_{\frac{a}{b}} b$$

3. II is true:

The graph $y=\log_a \frac{x}{c}$ and $y=\log_b \frac{x}{d}$ intersect at the same point on the x-axis.

Sub y = 0:

$0=\log_a \frac{x}{c}$ and $0=\log_b \frac{x}{d}$

$\frac{x}{c}=1$ and $\frac{x}{d}=1$

$x=c$ and $x=d$

Thus, $c=d$.

I is not true:

Add line y=1.

The x-coordinates of the intersecting point with $y=\log_a \frac{x}{c}$ and $y=\log_b \frac{x}{d}$ are ac and bd respectively.

From the graph, ac > bd.

Since c = d, a > b.

III is true:

Let C(k,0).

$B(k,\log_b \frac{k}{d})$

$A(k,\log_a \frac{k}{c})$

$$\frac{AC}{AB}=\frac{0-\log_a \frac{k}{c}}{\log_b \frac{k}{d}-\log_a \frac{k}{c}}$$

$$\frac{AC}{AB}=\frac{\frac{-\log_a \frac{k}{c}}{\log_a \frac{k}{c}}}{\frac{\log_b \frac{k}{d}}{\log_a \frac{k}{c}}-\frac{\log_a \frac{k}{c}}{\log_a \frac{k}{c}}}$$

$$\frac{AC}{AB}=\frac{-1}{\log_b a-1} \quad (\because c=d)$$

$$\frac{AC}{AB}=\frac{1}{\log_b b-\log_b a}$$

$$\frac{AC}{AB}=\frac{1}{\log_b \frac{b}{a}}$$

$$\frac{AC}{AB}=\log_{\frac{b}{a}} b$$