POLYNOMIALS, EXPONENTIAL AND LOGARITHMIC FUNCTION

Form 6

Vol 7

Part 7 – Index Simplification

1. (a) $\left(\dfrac{2ab^{-2}}{c^{-3}}\right)^{-4}$

$= \dfrac{2^{-4}a^{-4}b^{8}}{c^{12}}$

$= \dfrac{b^{8}}{16a^{4}c^{12}}$

(b) $\dfrac{(4p^{2}q)^{-1}(4p^{3}q^{-2})^{2}}{4(pq)^{-3}(p^{5}q^{-3})}$

$= \dfrac{(4^{-1}p^{-2}q^{-1})(4^{2}p^{6}q^{-4})}{4(p^{-3}q^{-3})(p^{5}q^{-3})}$

$= 4^{-1+2-1}p^{-2+6+3-5}q^{-1-4+3+3}$

$= p^{2}q$

(c) $\dfrac{(-15x^{3}y^{-6})(14x^{-2}y^{4})}{35x^{-5}y^{9}}$

$= -6x^{3-2+5}y^{-6+4-9}$

$= -\dfrac{6x^{6}}{y^{11}}$

(d) $\sqrt[5]{\dfrac{32ab^{4}}{243c^{10}}}$

$= \dfrac{2a^{\frac{1}{5}}b^{\frac{4}{5}}}{3c^{2}}$

(e) $\sqrt{x\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}}$

$= x^{\frac{1}{2}}\cdot x^{\frac{1}{4}}\cdot x^{\frac{1}{8}}\cdot x^{\frac{1}{16}}\cdot x^{\frac{1}{32}}$

$= x^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}}$

$= x^{\frac{31}{32}}$

2. (a) $\dfrac{27^n \cdot 16^{\frac{n}{2}}}{6^{2n}}$

$= \dfrac{3^{3n} \cdot 2^{2n}}{3^{2n} \cdot 2^{2n}}$

$= 3^{3n-2n} \cdot 2^{2n-2n}$

$= 3^n$

(b) $\dfrac{49^x \cdot 7^{x+1}}{7^{\frac{x}{2}}}$

$= \dfrac{7^{2x} \cdot 7^{x+1}}{7^{\frac{x}{2}}}$

$= 7^{2x+x+1-\frac{x}{2}}$

$= 7^{\frac{5}{2}x+1}$

(c) $\dfrac{3^n \cdot 8^n}{2^{2n}}$

$= \dfrac{3^n \cdot 2^{3n}}{2^{2n}}$

$= 3^n \cdot 2^{3n-2n}$

$= 3^n \cdot 2^n$

$= 6^n$

(d) $\dfrac{5^{2n} + 5^{2n-1}}{5^{2n}}$

$= \dfrac{5^{2n}(1+5^{-1})}{5^{2n}}$

$= \dfrac{6}{5}$

(e) $\dfrac{3^{n+2} - 27(3^{n-2})}{9(3^n)}$

$= \dfrac{3^n(3^2 - 27 \cdot 3^{-2})}{9(3^n)}$

$= \dfrac{2}{3}$

(f) $\dfrac{3\cdot 4^{3n-2}+5\cdot 8^{2n}}{4^{3n}+8^{2n-1}}$

$=\dfrac{3\cdot 2^{6n-4}+5\cdot 2^{6n}}{2^{6n}+2^{6n-3}}$

$=\dfrac{2^{6n}(3\cdot 2^{-4}+5)}{2^{6n}(1+2^{-3})}$

$=\dfrac{83}{18}$

Part 8 – Log Simplification

1. $\log\sqrt{a}-\log\sqrt[3]{a}$

$=\log a^{\frac{1}{2}}-\log a^{\frac{1}{3}}$

$=\dfrac{1}{2}\log a-\dfrac{1}{3}\log a$

$=\dfrac{1}{6}\log a$

2. $\dfrac{\log\sqrt{x}}{\log x^{\frac{2}{5}}+\log\dfrac{1}{x}}$

$=\dfrac{\log x^{\frac{1}{2}}}{\log x^{\frac{2}{5}-1}}$

$=\dfrac{\frac{1}{2}\log x}{-\frac{3}{5}\log x}$

$=-\dfrac{5}{6}$

3. $\dfrac{\log\sqrt{x}+1}{\log(100x)}$

$=\dfrac{\log x^{\frac{1}{2}}+1}{\log x+\log 100}$

$=\dfrac{\frac{1}{2}\log x+1}{\log x+2}$

$=\dfrac{\frac{1}{2}(\log x+2)}{\log x+2}$

$=\dfrac{1}{2}$

4. $\log(2ab)^2+\log\dfrac{5}{a^2}-\log\dfrac{\sqrt{b}}{5}$

$=\log\left[(2^2)(5^{1+1})a^{2-2}b^{2-\frac{1}{2}}\right]$

$=\log(100b^{\frac{3}{2}})$

$=2+\dfrac{3}{2}\log b$

5. $\dfrac{\log\sqrt[3]{a}-\log b}{\log(a^{-1}b^3)}$

$=\dfrac{\log(a^{\frac{1}{3}}b^{-1})}{\log(a^{-1}b^3)}$

$=\log_{a^{-1}b^3}(a^{-1}b^3)^{-\frac{1}{3}}$

$=-\dfrac{1}{3}$

6. $\dfrac{\log_{\frac{1}{4}}(a^2b^{-4})}{\log_8\sqrt{a}-\log_8(ab^{-1})}$

$=\dfrac{\log_8(a^2b^{-4})^{-\frac{3}{2}}}{\log_8(a^{\frac{1}{2}-1}b)}$

$=\log_{a^{-\frac{1}{2}}b}(a^{-3}b^6)$

$=\log_{a^{-\frac{1}{2}}b}(a^{-\frac{1}{2}}b)^6$

$=6$

7. Note that $12.5=\dfrac{25}{2}=\dfrac{5^2}{2}$.

$\log 12.5$

$=\log\dfrac{5^2}{2}$

$=\log 5^2-\log 2$

$=2\log 5-\log 2$

$=2b-a$

8. Note that $\sqrt{18}=3\sqrt{2}=3\cdot 2^{\frac{1}{2}}$.

$\log_n\sqrt{18}$

$=\log_n(3\cdot 2^{\frac{1}{2}})$

$=\log_n 3+\log_n 2^{\frac{1}{2}}$

$=\log_n 3+\dfrac{1}{2}\log_n 2$

$=b+\dfrac{1}{2}a$

Part 9A – Exp/log equations

1. $8^{x+2} = 16$

$2^{3x+6} = 2^4$

$3x+6=4$

$x = -\frac{2}{3}$

2. $5^x = \frac{1}{\sqrt{5}}$

$5^x = 5^{-\frac{1}{2}}$

$x = -\frac{1}{2}$

3. $4^{x-1} + 8 = 20$

$4^{x-1} = 12$

$\log_4 4^{x-1} = \log_4 12$

$x - 1 = \log_4 12$

$x = \log_4 12 + 1$

$x \approx 2.79$

4. $3(2^x) = 5^{x+1}$

$3(2^x) = 5(5^x)$

$\frac{2^x}{5^x} = \frac{5}{3}$

$\log_{\frac{2}{5}} \left(\frac{2}{5}\right)^x = \log_{\frac{2}{5}} \frac{5}{3}$

$x = \log_{\frac{2}{5}} \frac{5}{3}$

$x \approx -0.557$

5. $8^{x-2}=3^{x+3}$

$8^{-2}(8^x)=3^3(3^x)$

$\frac{8^x}{3^x}=1728$

$\log_{\frac{8}{3}}\left(\frac{8}{3}\right)^x=\log_{\frac{8}{3}}1728$

$x=\log_{\frac{8}{3}}1728$

$x\approx 7.60$

6. $2^{2x+3}-4^x=35$

$2^3(4^x)-4^x=35$

$4^x(2^3-1)=35$

$4^x=5$

$\log_4 4^x=\log_4 5$

$x=\log_4 5$

$x\approx 1.16$

7. $3^{x+1}-3^x-54=0$

$3(3^x)-3^x=54$

$3^x(3-1)=54$

$3^x=27$

$3^x=3^3$

$x=3$

8. $9^x-2(3^{2x-1})=27$

$9^x-2(3^{-1})(9^x)=27$

$9^x(1-2(3^{-1}))=27$

$9^x=81$

$x=2$

9. $3^{2x}-7(3^x)+6=0$

$(3^x-1)(3^x-6)=0$

$3^x=1$ or $3^x=6$

$3^x=3^0$ or $\log_3 3^x=\log_3 6$

$x=0$ or $x=\log_3 6$

$x\approx 1.63$

10. $16^x - 5(4^{x+1}) + 96 = 0$

$4^{2x} - 5(4)(4^x) + 96 = 0$

$4^{2x} - 20(4^x) + 96 = 0$

$(4^x - 8)(4^x - 12) = 0$

$4^x = 8$ or $4^x = 12$

$\log_4 4^x = \log_4 8$ or $\log_4 4^x = \log_4 12$

$x = \log_4 8$ or $x = \log_4 12$

$x = \dfrac{3}{2}$ or $x \approx 1.79$