COORDINATE GEOMETRY(III)

Form 6
Vol 6

Part 5B – 4 centre questions

1. $AB = \sqrt{(25-18)^2 + (0-24)^2}$
$= 25$
$= OB$

So, ΔOAB is an isosceles triangle.
Denote the incentre of ΔOAB by I.
Let H and K be the points lying on OA and OB respectively such that $IH \perp OA$ and $IK \perp OB$.

$OA = \sqrt{(18-0)^2 + (24-0)^2} = 30$

$OH = 15$ (prop. of isos. Δ)
$OK = 15$ (tangent prop.)
$BK = 25 - 15 = 10$

$\sin \angle OBH = \frac{15}{25} = \frac{3}{5}$

$\tan \angle OBH = \frac{3}{4}$

$IK = BK \tan \angle OBH$
$= 10 \times \frac{3}{4}$
$= \frac{15}{2}$

Note that IK is vertical.

Thus, the coordinates of the incentre of ΔOAB are $\left(15, \frac{15}{2}\right)$.

2. Note that AB is horizontal.

The vertical line $x=1$ passes through C.

Let the coordinates of C be $(1,k)$.

Note that the line passing through A and is perpendicular to BC passes through the orthocentre of ΔABC.

$$\frac{3-\frac{21}{5}}{2-1}\times\frac{k-3}{1+5}=-1$$

$k=8$

Thus, the coordinates of C are $(1,8)$.

3. Note that the line $x+5y=a$ bisects $\angle OAB$.

$$\angle OAB=2\tan^{-1}\left(\frac{1}{5}\right)$$

$$\tan\angle OAB=\frac{OB}{OA}$$

$$\frac{5}{12}=\frac{b}{a}$$

Thus, we have $a:b=12:5$.

4. Note that AB is horizontal and $\angle OAB=90^\circ$.

The coordinates of the incentre of ΔOAB are $I(1,a-1)$.

Slope of $BI=\dfrac{a-(a-1)}{3-1}=\dfrac{1}{2}$

$$\tan\frac{\angle ABO}{2}=\frac{1}{2}$$

$$\tan\angle ABO=\frac{4}{3}$$

Slope of $OB=\dfrac{4}{3}$

$$\frac{a}{3}=\frac{4}{3}$$

$\therefore\ a=4$

Alternatively,

Denote the incentre of ΔOAB by I.

Let H, M and N be the points lying on OA, OB and AB respectively such that $IH \perp OA$, $IM \perp OB$ and $IN \perp AB$.

Note that $AH = AN = 1$.

$OM = OH$ (tangent prop.)

$= a - 1$

$MB = NB$ (tangent prop.)

$= 3 - 1$

$= 2$

$OB = OM + MB$

$= a + 1$

So, we have

$a + 1 = \sqrt{a^2 + 3^2}$

$2a = 8$

$\therefore \ a = 4$

5. Slope of $AB = \frac{18+6}{-6+13} = \frac{24}{7}$

Slope of $BC = -\frac{4}{3}$

$\angle ABC = 180° - \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{4}{3}\right)$

Slope of the angle bisector of $\angle ABC = \tan\left[\tan^{-1}\left(\frac{24}{7}\right) + \frac{\angle ABC}{2}\right] = -\frac{11}{2}$

The equation of the angle bisector of $\angle ABC$ is

$y - 18 = -\frac{11}{2}(x+6)$

$y = -\frac{11}{2}x - 15$

$\begin{cases} 3x - 4y = 15 \\ y = -\frac{11}{2}x - 15 \end{cases}$

Solving, we have $x = -\frac{9}{5}$, $y = -\frac{51}{10}$.

Thus, the coordinates of the incentre of ΔABC are $\left(-\frac{9}{5}, -\frac{51}{10}\right)$.

6. Note that L_1 and L_2 intersect at the point $(0,2)$. Denote it by C.
Suppose the line $y = a$ intersects L_1 and L_2 at the points A and B respectively.
Then, the triangle formed by L_1, L_2 and the line $y = a$ is ΔABC.

$\angle CAB = \tan^{-1}\left(\frac{12}{5}\right)$

Note that ΔABC is an isosceles triangle. So, the incentre lies on the y-axis.
The coordinates of the incentre of ΔABC are $(0,15)$.

The distance from A to the y-axis is $\frac{5a-10}{12}$.

So, we have

$\tan\left(\frac{\angle CAB}{2}\right) = \frac{a-15}{\frac{5a-10}{12}}$

$\frac{2}{3} = \frac{12a-180}{5a-10}$

$a = 20$

7. Note that $2(5)-3(0)-10=0$.

So, A lies on the line $2x-3y-10=0$.

Thus, slope of $AB=\tan\left[2\tan^{-1}\left(\frac{2}{3}\right)\right]=\frac{12}{5}$.

Let the coordinates of B be $(0,b)$.

$\frac{b-0}{0-5}=\frac{12}{5}$

$b=-12$

The coordinates of B are $(0,-12)$.