超研數學 ASPIRE MATHEMATICS

SOLUTION

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TRANSFORMATION OF GRAPH

Form 5

Vol 8

Part 1 – Translation

1. B	2. B	3. C	4. D	5. A	6. B	7. D
8. B	9. A	10. A				

1. The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ downwards by 2 units.
   Thus, we have $g(x) = f(x) - 2$.

2. The graph of $y = f(x) + 1$ is obtained by translating the graph of $y = f(x)$ upwards by 1 unit.
   Thus, B is the desired answer.

3. We can observe that the graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ upwards by k units and rightwards by h units.
   Then, the coordinates of the vertex of the graph of $y = g(x)$ are $(7 + h, -11 + k)$.
   Note that the graph of $y = f(x)$ passes through the origin.
   So, we have 14 is an x-intercept of the graph of $y = f(x)$.
   Hence, we have $h = 7$.
   Since the vertex of the graph $y = g(x)$ touches the x-axis, we have $k = 11$.
   Thus, we have $g(x) = f(x - 7) + 11$.

4. The coordinates of the vertex of the graph of $y = f(x)$ are $(-1, 0)$.
   Observe that the graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ downwards by k units and leftwards by h units.
   Then, the coordinates of the vertex of the graph of $y = g(x)$ are $(-1 - h, -k)$.
   Since $-1 - h = -4$, we have $h = 3$.
   So, we have $g(x) = f(x + 3) - k$.
   Since the graph of $y = g(x)$ passes through $(-1,0)$, we have
   $f(2) - k = 0$
   $(2)^2 + 2(2) + 1 - k = 0$
   $k = 9$
   Thus, we have $g(x) = f(x + 3) - 9$.

5. Let $f(x) = 7x^2 + x - 10$.

The required function is:

$y = f(x-5) - 8$

$y = 7(x-5)^2 + (x-5) - 10 - 8$

$y = 7x^2 - 69x + 152$

6. Let $g(x) = x^2 + x + 5$.

The required function is

$y = g(x-2) - 3$

$y = (x-2)^2 + (x-2) + 5 - 3$

$y = x^2 - 3x + 4$

7. The transformed function is $y = \log_4(x+3) - 20$.

Since $x + 3 > 0$, we have $x > -3$.

Thus, the required domain is all real numbers greater than – 3.

8. Let $f(x) = -3x^2 - 12x - 25$.

Then, we have $f(x) = -3(x+2)^2 - 13$.

$y = -3x^2 + 30x - 105$

$= -3(x-5)^2 - 30$

$= -3[(x-7)+2] - 13 - 17$

Thus, the transformed graph is obtained by translating the graph of $y = f(x)$ to the right by 7 units and 17 units downwards.

9. Let $f(x) = 2x^2 - 20x + 60$.

Then, we have $f(x) = 2(x-5)^2 + 10$.

$y = 2x^2 - 4x + 7$

$= 2(x-1)^2 + 5$

$= 2[(x+4)-5] + 10 - 5$

Thus, the transformed graph is obtained by translating the graph of $y = f(x)$ to the left by 4 units and 5 units downwards.

10. $g(x) = f(x+5)$

$= 2(x+5)^2 + 10(x+5) - 72$

$= 2x^2 + 30x + 28$

By considering $g(x) = 0$, we have $x = -14$ or $x = -1$.

Thus, the x-intercepts of the graph of $y = g(x)$ are -14 or -1.

Part 2A – Reflection

1. C	2. C	3. A	4. A	5. B	6. A	7. C
8. D	9. D	10. A	11. B	12. B		

1. Let $g(x) = f(-x) + 3$.
   Note that the graph of $y = f(x)$ passes through $(0, -1)$.
   So, we have $g(0) = f(0) + 3 = 2$.
   Thus, the y-intercept of the graph of $y = g(x)$ is 2.
   Hence, the required y-intercept is 2.

2. Let $f(x) = 3^x$.
   The required function is
   $y = f(-x) + 1$
   $y = 3^{-x} + 1$
   $y = \frac{1}{3^x} + 1$

3. Let $f(x) = 4x^2 + x - 9$.
   The required function is
   $y = -[f(x) - 4]$
   $y = -(4x^2 + x - 13)$
   $y = -4x^2 - x + 13$

4. Let $f(x) = -3x^2 - 10x + 7$.
   The required function is
   $y = f[-(x + 2)]$
   $y = -3[-(x + 2)]^2 + 10(x + 2) + 7$
   $y = -3x^2 - 2x + 15$

5. $y = f(x) - 4$
   $= -[-f(x) + 6 - 2]$
   Thus, the graph of $y = f(x) - 4$ is obtained by translating the graph of $y = -f(x) + 6$ downwards by 2 units and then reflect the resulting graph along the x-axis.

6. $y = f(-x + 3)$
   $= f[(-x) - 5 + 8]$
   Thus, the graph of $y = f(-x + 3)$ is obtained by translating the graph of $y = f(x - 5)$ to left by 8 units and then reflect the resulting graph along to the y-axis.

7. Let $f(x) = 2x^2 - 12x + 1$.
Then, we have $f(x) = 2(x - 3)^2 - 17$.

$y = 2x^2 - 8x - 9$
$= 2(x - 2)^2 - 17$
$= 2[-(-x + 2)]^2 - 17$
$= 2(-x + 2)^2 - 17$
$= 2[(-x) - 3 + 5]^2 - 17$
$= 2[-(x - 5) - 3]^2 - 17$

Thus, the graph of $y = 2x^2 - 8x - 9$ is obtained by reflecting the graph of $y = f(x)$ along the y-axis and then translated the resulting graph to the right by 5 units.

8. The graph of $y = -3 - f(x)$ is obtained by reflecting the graph of $y = f(x)$ along the x-axis and then translated the resulting graph downwards by 3 units.
Thus, D is the desired answer.

9. The graph of $y = f(1 - x)$ is obtained by reflecting the graph of $y = f(x)$ along the y-axis and then translating the resulting graph to the right by 1 unit.
Thus, D is the desired answer.

10. Suppose that the graph of $y = f(x)$ passes through the points (1, 0) and (4, 0).
Then, the transformed function is obtained by reflecting the graph of $y = f(x)$ along the y-axis and then translating to the left by 1 unit.
Hence, the transformed function is $y = f[-(x + 1)]$.
Thus, the figure shows the graph of $y = f(x)$ and the graph of $y = f(-1 - x)$.

11. Note that the y-intercept of the graph of $y = a^{2-x}$ is a^2.
Since $a > 1$, we have $a^2 > 1$ and $a^{-x} < 1$.
Hence as x increases, y decreases.
Thus, B is the desired answer.

12. Let $f(x) = \log_a x$.
Note that the figure shows the transformed graph from the graph of $y = f(x)$.
Consider $0 < a < 1$, observe that $f(x) > 0$ as $0 < x < 1$ and $a^2 < 1$.
So, the transformed graph may be obtained by reflecting the graph of $y = f(x)$ along the x-axis and then translating the resulting graph upwards by k units.
Hence, the transformed function is in the form of $y = -f(x) + k$.
Since the graph of the transformed function passes through $(a^2, 0)$, we have $-f(a^2) + k = 0$.
Therefore, we have $k = 2$.
Thus, the figure shows the graph of the function $y = -\log_a x + 2$.