

REGULAR QUIZ 03

Form 6

Coordinate geometry (II), (III)

Part A – MC (@3 marks)

1.	A	<p>Let $f(x) = 3^x$</p> <p>After reflection, $-f(x) = -3^x$</p> <p>After compression, $-f(2x) = -3^{2x}$</p> <p>After translation, $-f(2(x+2)) = -3^{2x+4}$</p>
2.	C	<p>Since $f(0) = -3$, $f(2) = f(4) = 0$</p> <p>Then $g(0) = -\frac{1}{2}f(0) = \frac{3}{2}$, $g(2) = -\frac{1}{2}f(2) = 0$ and $g(4) = -\frac{1}{2}f(4) = 0$</p> <p>$\therefore g(x)$ passes through $\left(0, \frac{3}{2}\right)$, $(2, 0)$ and $(4, 0)$</p>
3.	A	<p>Maximum and minimum of the graph are 0 and -4 respectively and it attains its maximum when $x = 40$.</p> <p>$\therefore y = 2\sin(x^\circ + 50^\circ) - 2$</p>
4.	B	<p>Only II is incorrect. Its locus is perpendicular bisector of AE.</p>
5.	B	<p>Equation of P</p> $AP = PB$ $(x-2)^2 + (y+3)^2 = (x+4)^2 + (y-1)^2$ $-4x+4+6y+9 = 8x+16-2y+1$ $12x-8y+4 = 0$ $3x-2y+1 = 0$
6.	B	<p>Note that $L_1 // L_2$ with slope $-\frac{3}{4}$</p> <p>y-int of $L_1 = \frac{5}{4}$</p> <p>y-int of locus of $P = \frac{13}{16}$</p> <p>Equation of L_2</p> $y = -\frac{3}{4}x + \frac{13}{16} + \left(\frac{13}{16} - \frac{5}{4}\right)$ $y = -\frac{3}{4}x + \frac{3}{8}$ $6x + 8y - 3 = 0$

7.	B	<p>Slope of $AB = 2$ y-int of $AB = -1$ Let L be a straight line parallel to AB and passes through C. Then y-int of L $\frac{y-2}{0-6} = 2$ $y = -10$ \therefore The locus of P $y = 2x - 1 + (-1 + 10)$ $2x - y + 8 = 0$</p>
8.	A	<p>$x^2 + y^2 - 2x + 6ky - 37.5 = 0$ Centre $G(1, -3k)$ $r^2 = 1^2 + (-3k)^2 + 37.5 = 9k^2 + 38.5$ $AG^2 = (2k - 1)^2 + (3 - 2k + 3k)^2 = 5k^2 + 2k + 10$ Consider $r^2 - AG^2$ $= 4k^2 - 2k + 28.5$ $= 4\left(k - \frac{1}{4}\right)^2 + \frac{113}{4}$ Since $\left(k - \frac{1}{4}\right)^2 \geq 0$, $\therefore r^2 - AG^2 > 0 \Rightarrow r > AG$ A must lie inside C. I is true. If $MN = 18$, $1^2 + \left(\frac{18}{2}\right)^2 = r^2$ $9k^2 + 38.5 = 82$ $k^2 = \frac{29}{6}$ II is false. Minimum area of the circle $= 38.5\pi$ III is false.</p>
9.	D	<p>Centre (h, k), radius $= k$ and $0 < h < k$ \therefore centre lies in quadrant I with x-axis is its tangent.</p>

10.	A	<p>Let centre be (h, r)</p> $\sqrt{(h-3)^2 + (r-1)^2} = r$ $(h-3)^2 + r^2 - 2r + 1 = r^2$ $(h-3)^2 + 1 = 2r$ $(h-3)^2 + 1 > 26$ $(h-3)^2 > 25$ $h-3 < -5 \quad \text{or} \quad h-3 > 5$ $h < -2 \quad \text{or} \quad h > 8$
11.	A	<p>Perpendicular bisector of AB</p> $(x-8)^2 + (y-5)^2 = (x-4)^2 + (y+3)^2$ $x+2y-8=0$ $\begin{cases} x+2y-8=0 \\ 3x-4y+6=0 \end{cases}$ <p>\therefore Centre $(2, 3)$</p> <p>Equation of circle</p> $(x-2)^2 + (y-3)^2 = (8-2)^2 + (5-3)^2$ $x^2 + y^2 - 4x - 6y - 27 = 0$
12.	C	<p>Denote centre be $C(-1, 2)$ and the mid-point of AB be M.</p> <p>Equation of MC</p> $y-2 = \frac{1}{3}(x+1)$ $x-3y+7=0$ $\begin{cases} 3x+y-k=0 \\ x-3y+7=0 \end{cases}$ $x = \frac{3k-7}{10}$
13.	A	$\begin{cases} x+y+m=0 \\ (x+1)^2 + (y-3)^2 = 8 \end{cases}$ $(x+1)^2 + (-x-m-3)^2 = 8$ $2x^2 + (2m+8)x + m^2 + 6m + 2 = 0$ $\Delta \geq 0$ $(2m+8)^2 - 4(2)(m^2 + 6m + 2) \geq 0$ $-4m^2 - 16m + 48 \geq 0$ $\therefore -6 \leq m \leq 2$

1. A 2. C 3. A 4. B 5. B 6. B 7. B
 8. A 9. D 10. A 11. A 12. C 13. A

Part B - Long Questions

1. (8 marks)

(a) $f(x)$

$$= 4x^2 - 12x + k + 7$$

$$= 4\left(x - \frac{3}{2}\right)^2 + k - 2 \quad 1M$$

$$\therefore U\left(\frac{3}{2}, k - 2\right) \quad 1A$$

(b)(i) $g(x)$

$$= -x^2 + 6x - k - 3$$

$$= -(x - 3)^2 - k + 6$$

$$\therefore V(3, -k + 6)$$

1A

$f(x)$ is reflected about the x -axis, translated upwards by 4 units
 and enlarged to 2 times along the x -axis

1A

+1A

(b)(ii) Note that $OU \perp UV$.

1M

$$\frac{k-2}{\frac{3}{2}} \times \frac{k-2-(-k+6)}{\frac{3}{2}-3} = -1$$

1M

$$8k^2 - 48k + 55 = 0$$

$$k = \frac{48 \pm \sqrt{48^2 - 4(8)(55)}}{2(8)}$$

$$k = 3 + \frac{\sqrt{34}}{4} \quad \text{or} \quad k = 3 - \frac{\sqrt{34}}{4} \quad (\text{r.t. } k = 4.46 \quad \text{or} \quad k = 1.54) \quad 1A$$

2. (10 marks)

(a) Let the equation of C be $x^2 + y^2 + Dx + Ey + F = 0$

Put $(0,0)$, we have $F = 0$

1A

Put $A(2,4)$, $2D + 4E + 20 = 0$

Put $B(6,7)$, $6D + 7E + 85 = 0$

Solving, $D = -20$, $E = 5$

1A for either one

The equation of C

$$x^2 + y^2 - 20x + 5y = 0$$

1A

(b) Let the equation of tangent be $y - 3 = m(x - 22)$

i.e. $y = mx - 22m + 3$

$$x^2 + (mx - 22m + 3)^2 - 20x + 5(mx - 22m + 3) = 0$$

1M

$$(1 + m^2)x^2 + (-44m^2 + 11m - 20)x + 484m^2 - 242m + 24 = 0$$

$$\Delta = 0$$

$$(-44m^2 + 11m - 20)^2 - 4(1 + m^2)(484m^2 - 242m + 24) = 0$$

1M

$$-151m^2 + 528m + 304 = 0$$

$$m = 4 \text{ or } m = -\frac{76}{151}$$

The equation of the tangents

$$4x - y - 85 = 0 \quad 76x + 151y - 2125 = 0$$

1A+1A

(c) Denote $4x - y - 85 = 0$ and $76x + 151y - 2125 = 0$ be L_1
and L_2

Inclination of L_1

$$\tan \theta_1 = 4$$

$$\theta_1 = 75.96375653$$

Inclination of L_2

$$\tan \theta_2 = -\frac{76}{151}$$

$$\theta_2 = 153.283373$$

$$\angle KVM = \frac{1}{4}(\theta_2 - \theta_1) = 19.3^\circ$$

1M for $\theta_2 - \theta_1$

or finding $\angle UKV = \theta_1 + 180^\circ - \theta_2$

alternatively

+1M for $\frac{1}{4}$

+1A

3. (12 marks)

(a) Let centre be $(4, r)$

$$\sqrt{(4-0)^2 + (r-8)^2} = r$$

$$r = 5 \quad 1A$$

Equation of C

$$(x-4)^2 + (y-5)^2 = 25 \quad 1A$$

(b) Slope of $L_1 = -\frac{3}{4}$ 1M

Equation of L_2

$$y-5 = \frac{4}{3}(x-4) \quad 1M$$

$$4x-3y-1=0 \quad 1A$$

(c)(i) $\begin{cases} 3x+4y+18=0 \\ 4x-3y-1=0 \end{cases}$
 $S(-2, -3)$ 1A

(c)(ii) Shortest distance

$$= \sqrt{(4+2)^2 + (5+3)^2} - 5 \quad 1M \text{ for } \sqrt{(4+2)^2 + (5+3)^2}$$

+1M for -5

$$= 5 \quad 1A$$

(c)(iii) Centre of the required circle

$$= \left(\frac{4+3(-2)}{1+3}, \frac{5+3(-3)}{1+3} \right) \quad 1M$$

$$= \left(-\frac{1}{2}, -1 \right)$$

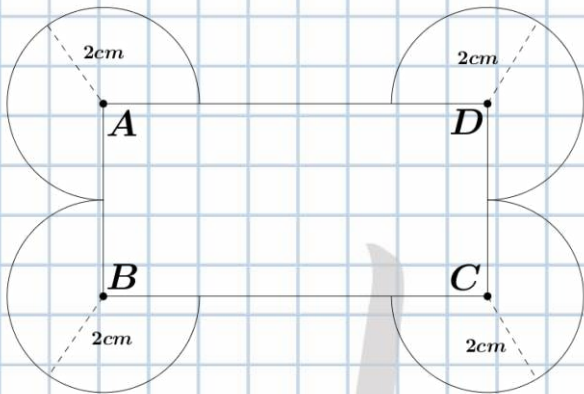
Equation of the required circle

$$\left(x + \frac{1}{2} \right)^2 + (y+1)^2 = \left(\frac{5}{2} \right)^2 \quad 1M \text{ for } \frac{5}{2}$$

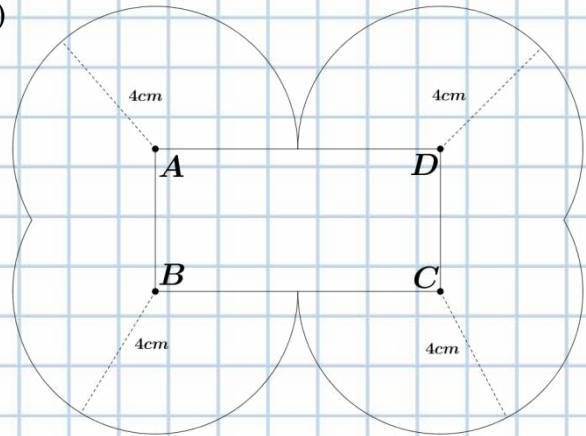
$$x^2 + y^2 + x + 2y - 5 = 0 \quad +1A$$

4. (Bonus) (9 marks)(3@)

(a)



(b)



(c)

