REGULAR QUIZ 04

Form 5

Equation of Circle

Part A – MC (@2 marks)

1.	C	The straight line passes through the centre of circle. Centre $(2,4)$ $3(2)-2(4)+p=0$ $p=2$
2.	D	Centre $\left(\frac{6+0}{2},\frac{0-8}{2}\right)=(3,-4)$ and the circle passes through origin. $\therefore D=-6,\ E=8,\ F=0$ $x^2+y^2-6x+8y=0$
3.	B	I is incorrect. Put $(-1,1)$ $4(-1)^2+4(1)^2+16(-1)-36(1)+45=1\neq 0$ II is incorrect. Radius $=\sqrt{\left(\frac{16}{-8}\right)^2+\left(\frac{-36}{-8}\right)^2-\left(\frac{45}{4}\right)}=\sqrt{13}\neq 7\sqrt{7}$ III is correct Put $x=0$ $4y^2-36y+45=0$ $\Delta=36^2-4(4)(45)=576>0$ $\therefore$ intersects 2 distinct points
4.	A	Radius = 7 Equation of circle $(x-3)^2+(y+7)^2=7^2$ $x^2+y^2-6x+14y+9=0$
5.	B	Centre $(1,-1.5)$ Distance < radius $1^2+1.5^2<1^2+1.5^2-F$ $F<0$

6.	A	Let the centre be $(h, 2h+1)$ $(h-1)^2+(2h+1-0)^2=(h-4)^2+(2h+1-3)^2$ $-2h+1+4h+1=-8h+16-8h+4$ $18h=18$ $h=1$ Centre $(1,3)$ Equation of circle $(x-1)^2+(y-3)^2=(1-1)^2+(0-3)^2$ $x^2+y^2-2x-6y+1=0$
7.	B	Centre $(2.5,3)$ Put $y=0$ $x^2-5x+4=0$ $x=1$ or $x=4$ Area of $\Delta CAB=\frac{(4-1)\times 3}{2}=\frac{9}{2}$ sq. units
8.	C	Centre $(-3,2)$ Equation of straight line passes through centre and perpendicular to $x-y-k=0$ $y-2=-(x+3)$ $x+y+1=0$ $\begin{cases} x-y-k=0 \\ x+y+1=0 \end{cases}$ Solving, $\left(\frac{k-1}{2}, \frac{-k-1}{2}\right)$
9.	B	Put $A(0,4)$ $0^2+4^2+6(0)+k(4)-16=0$ $k=0$ $\therefore$ Centre $(-3,0)$ Slope of tangent $=-\frac{3}{4}$ Equation of tangent $y-4=-\frac{3}{4}x$ $3x+4y-16=0$

10.	B	$AM = \sqrt{(0-4)^2 + (2+2)^2} = \sqrt{32}$ $BM = \sqrt{(0-4)^2 + (-6.5+2)^2} = \sqrt{36.25}$ $AB = 8.5$ $\cos \angle AMB = \dfrac{AM^2 + BM^2 - 8.5^2}{2(AM)(BM)}$ $\angle AMB \approx 93.36646066^\circ$ $\therefore$ an obtuse angle
11.	A	$x = 2y - k$ $(2y-k)^2 + y^2 - 24(2y-k) - 10y + 89 = 0$ $4y^2 - 4ky + k^2 + y^2 - 48y + 24k - 10y + 89 = 0$ $5y^2 + (-4k-58)y + k^2 + 24k + 89 = 0$ $\Delta > 0$ $(-4k-58)^2 - 4(5)(k^2 + 24k + 89) > 0$ $4k^2 + 116k + 841 - 5k^2 - 120k - 445 > 0$ $-k^2 - 4k + 396 > 0$ $\therefore -22 < k < 18$

1.	C	2.	D	3.	B	4.	A	5.	B
6.	A	7.	B	8.	C	9.	B	10.	B
11.	A								

Part B - Short Questions (18 marks)

1. (5 marks)

(a) Put $(5.5, 0)$ into L

$5.5k = 22$

$k = 4$ 1A

(b) Centre of circle $(9, 7)$ 1M

Let $P\left(h, \frac{22-4h}{3}\right)$

$\frac{\frac{22-4h}{3}-7}{h-9} \times -\frac{4}{3} = -1$ 1M

$P\left(\frac{17}{5}, \frac{14}{5}\right)$

Shortest distance

$= \sqrt{\left(\frac{17}{5}-9\right)^2 + \left(\frac{14}{5}-7\right)^2} - \sqrt{17}$ 1M

$= 7 - \sqrt{17}$ 1A (r.t. 2.88)

2. (13 + 3 marks)

(a) Put $x=0, y=0, F=0$. 1A

$-\frac{D}{2}=6$

$D=-12$ 1A

$\frac{-E}{2}=8$

$E=-16$ 1A

(b) Let $y=\frac{3}{4}x+c$ be the equations of tangents. 1M

$\begin{cases} y=\frac{3}{4}x+c \\ x^2+y^2-12x-16y=0 \end{cases}$

$x^2+(\frac{3}{4}x+c)^2-12x-16(\frac{3}{4}x+c)=0$ 1M

$x^2+\frac{9}{16}x^2+\frac{3}{2}cx+c^2-12x-12x-16c=0$

$16x^2+9x^2+24cx+16c^2-384x-256c=0$

$25x^2+(24c-384)x+16c^2-256c=0$

$\Delta=0$

$(24c-384)^2-4(25)(16c^2-256c)=0$ 1M

$-1024c^2+7168c+147456=0$

$c=-9$ or $c=16$

$\therefore y=\frac{3}{4}x-9$ or $y=\frac{3}{4}x+16$ 1A+1A

(c) $L_1 : y = \frac{3}{4}x + 16$, $L_2 : y = \frac{3}{4}x - 9$

By solving L_1 and C, $A = (0,16)$. 1A

By solving L_2 and C, $X = (12,0)$. 1A

Put $y = 0$, $x = -\frac{64}{3}$.

$B = (-\frac{64}{3}, 0)$

$Y = (0,-9)$

Note that AX is a diameter.

Area of $ABYX$

$= \frac{1}{2}(\sqrt{(16-0)^2 + (-\frac{64}{3}-0)^2} + \sqrt{(-9-0)^2 + (0-12)^2})(10 \times 2)$ 2M

$= \frac{1250}{3}$

≈ 417 1A

(d) $\angle BAX = \angle AXY = 90°$

Slope of BY

$= \frac{9}{-\frac{64}{3} - 0} = -\frac{27}{64}$

Slope of XY

$= \frac{9}{12} = \frac{3}{4}$

Slope of $BY \times$ Slope of XY

$= -\frac{27}{64} \times \frac{3}{4}$

$= -\frac{81}{256}$

$\neq -1$ 1M

Therefore, BY is not perpendicular to XY.

$\angle BYX \neq 90°$

$\angle BYX + \angle BAX \neq 180°$ 1M

Thus, $ABYX$ is not concyclic. 1A f.t.