COORDINATE GEOMETRY(III)

Form 6
Vol 6

Part 1 – Equation of circle

1.	C	2.	B	3.	B	4.	A	5.	B
6.	A	7.	D	8.	C	9.	B	10.	B

1. C

The coordinates of the centre of the circle are $(2,4)$.

Note that the line $3x-2y+p=0$ passes through the centre of the circle.

Putting $x=2,\ y=4$ in $3x-2y+p=0$, we have

$3(2)-2(4)+p=0$

$p=2$

2. B

Since the origin lies inside the circle, we have

$(0)^2+(0)^2-2(0)+3(0)+F<0$

$F<0$

3. B

The coordinates of C are $\left(\frac{5}{2},3\right)$.

Putting $y=0$ in $x^2+y^2-5x-6y+4=0$, we have

$x^2-5x+4=0$

$x=1$ or $x=4$

So, the distance between A and B is 3.

The area of ΔABC

$=\frac{(3)(3)}{2}$

$=\frac{9}{2}$

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4. A

Putting $y=0$ into the equation of the circle, we have

$3x^2-22x+39=0$

$x=3$ or $x=\dfrac{13}{3}$

Putting $x=0$ into the equation of the circle, we have

$3y^2+42y+39=0$

$y=-13$ or $y=-1$

So, the coordinates of P, R and S are $(3,0)$, $(0,-1)$ and $(0,-13)$ respectively.

The area of ΔPRS

$=\dfrac{(-1+13)(3)}{2}$

$=18$

5. B

The coordinates of the centre of the circle are $(3,4)$.

Note that $(3,4)$ is the mid-point of AB.

The coordinates of B

$=(3\times 2-3, 4\times 2-5)$

$=(3,3)$

6. A

I is not true:

The radius of C $=\sqrt{\left(\dfrac{\frac{24}{3}}{2}\right)^2+\left(-\dfrac{\frac{28}{3}}{2}\right)^2-\dfrac{90}{3}}=\dfrac{\sqrt{70}}{3}$.

The area of C $=\pi\left(\dfrac{\sqrt{70}}{3}\right)^2\approx 24.43460953>24$.

II is true:

Note that $3(2)^2+3(-3)^2-24(2)+28(-3)+90=-3<0$.

The point $(2,-3)$ lies inside C.

III is not true:

The coordinates of the centre of C are $\left(4,-\dfrac{14}{3}\right)$,

which shows that the centre lies in the fourth quadrant.

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7. D

I is not true:

The radius of the circle $= \sqrt{\left(-\frac{\frac{30}{2}}{2}\right)^2 + \left(\frac{\frac{42}{2}}{2}\right)^2 - \frac{205}{2}} = 8$

II is true:

The coordinates of the centre of the circle are $\left(-\frac{15}{2}, \frac{21}{2}\right)$.

Note that $7\left(-\frac{15}{2}\right) + 5\left(\frac{21}{2}\right) = 0$.

Thus, the centre lies on the straight line $7x + 5y = 0$.

III is true:

The perpendicular distance between the centre and the x-axis

$= \frac{21}{2}$

> 8

So, the circle lies above the x-axis.

Thus, the circle does not intersect the x-axis.

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8. C

I is true:

Observe that the origin lies on C_1.

The coordinates of G_2 are $\left(-\frac{8}{5},-\frac{6}{5}\right)$.

Note that $\left(-\frac{8}{5}\right)^2+\left(-\frac{6}{5}\right)^2-14\left(-\frac{8}{5}\right)+22\left(-\frac{6}{5}\right)=0$.

So, G_2 lies on C_2.

Both OG_1 and G_1G_2 are radii of C_1.

Thus, ΔOG_1G_2 is an isosceles triangle.

II is not true:

$$G_1G_2=\sqrt{\left(\frac{14}{2}\right)^2+\left(-\frac{22}{2}\right)^2}=\sqrt{170}$$

The radius of C_2

$$=\sqrt{\left(-\frac{8}{5}\right)^2+\left(-\frac{6}{5}\right)^2+\left(\frac{825}{5}\right)}$$

$=13$

$< G_1G_2$

So, G_1 lies outside C_2.

Thus, the line segment OG_1 does not lie inside C_2.

III is true:

Since G_2 lies on C_1 and the radius of $C_2=13<G_1G_2$,

C_1 and C_2 intersect at two distinct points.

9. B

The coordinates of the centre of the circle $=\left(-\frac{2k}{2},\frac{10}{2}\right)=(-k,5)$.

So, we have

$\frac{-7-5}{4+k}=-12$

$k=-3$

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10. B

Denote the centres of the circles $(x-2)^2+(y+2)^2=100$ and $(x+10)^2+(y-7)^2=625$ by G_1 and G_2 respectively.

Note that $G_1P=10$ and $G_2P=25$.

So, $G_2G_1:G_1P=3:2$.

The coordinates of P

$=\left(\frac{(5)(2)-(2)(-10)}{3},\frac{(5)(-2)-(2)(7)}{3}\right)$

$=(10,-8)$

Part 2 – Condition of equation of circle

1.	D	2.	A	3.	D	4.	A	5.	D
6.	B	7.	B	8.	C	9.	D	10.	C
11.	A	12.	A						

1. D

The coordinates of the centre of the circle $=\left(\frac{4+2}{2},\frac{7-3}{2}\right)=(3,2)$

The radius of the circle $=\frac{\sqrt{(2-4)^2+(-3-7)^2}}{2}=\sqrt{26}$

The equation of the circle is

$(x-3)^2+(y-2)^2=26$

$x^2+y^2-6x-4y-13=0$

2. A

Let the coordinates of the centre of the circle be $(h,0)$.

$(h-5)^2+(0-10)^2=(h+1)^2+(0-8)^2$

$h=5$

The equation of the circle is

$(x-5)^2+y^2=(5-5)^2+10^2$

$x^2+y^2-10x-75=0$

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3. D

Denote the points $(3,7)$ and $(-4,6)$ by A and B respectively.

The coordinates of the mid-point of $AB = \left(\frac{3-4}{2}, \frac{7+6}{2}\right) = \left(-\frac{1}{2}, \frac{13}{2}\right)$

Slope of $AB = \frac{6-7}{-4-3} = \frac{1}{7}$

Note that the line joining from the centre to the mid-point of AB is perpendicular to AB.

So, we have $\frac{2-\frac{13}{2}}{k+\frac{1}{2}} = -7$.

$\therefore\ k = \frac{1}{7}$

The equation of the circle is

$$\left(x-\frac{1}{7}\right)^2 + (y-2)^2 = \left(3-\frac{1}{7}\right)^2 + (7-2)^2$$

$$x^2 + y^2 - \frac{2}{7}x - 4y - \frac{204}{7} = 0$$

$$7x^2 + 7y^2 - 2x - 28y - 204 = 0$$

4. A

Since the circle touches the x-axis, the radius of the circle is b.

The equation of the circle is

$$(x-a)^2 + (y-b)^2 = b^2$$

$$x^2 + y^2 - 2ax - 2by + a^2 = 0$$

5. D

6. B

Let the coordinates of the centre of the circle be $(-3, k)$.

$$(-3+7)^2 + (k-2)^2 = k^2$$

$$k = 5$$

The equation of the circle is

$$(x+3)^2 + (y-5)^2 = 5^2$$

$$x^2 + y^2 + 6x - 10y + 9 = 0$$

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7. B

Let the coordinates of the centre of the circle be $(-r, r)$.

Note that $\tan \angle ABO = \frac{2}{3}$.

$$\frac{r}{-r+15} = \frac{2}{3}$$

$r = 6$

So, the coordinates of the centre of the circle are $(-6, 6)$ and the radius of the circle is 6.

The equation of the circle is

$(x+6)^2 + (y-6)^2 = 36$

$x^2 + y^2 + 12x - 12y + 36 = 0$

8. C

Let the coordinates of the centre of the circle be (r, r).

Suppose AB touches the circle at the point R.

Then $AR = 8 - r$ and $BR = 6 - r$.

$AB = \sqrt{8^2 + 6^2} = 10$

So, we have $8 - r + 6 - r = 10$.

$\therefore\ r = 2$

The equation of the circle is

$(x-2)^2 + (y-2)^2 = 4$

$x^2 + y^2 - 4x - 4y + 4 = 0$

9. D

Denote the points $(2, 5)$ and $(6, 9)$ by A and B respectively.

Note that AB is a chord of the circle.

So, the centre of the circle lies on the perpendicular bisector of AB.

Slope of $AB = \frac{9-5}{6-2} = 1$

The coordinates of the mid-point of $AB = \left(\frac{2+6}{2}, \frac{5+9}{2}\right) = (4, 7)$.

The equation of the perpendicular bisector of AB is

$y - 7 = -(x - 4)$

$x + y - 11 = 0$

Therefore, the centre of the circle lies on the line $x + y - 11 = 0$.

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10. C

The radius of the circle $= \sqrt{3^2 + \left(\frac{8}{2}\right)^2} = 5$

The equation of the circle is

$(x-3)^2 + (y-3)^2 = 25$

$x^2 + y^2 - 6x - 6y - 7 = 0$

11. A

Note that $\angle OAB = 90°$.

So, the centre of the circle lies on AB.

The coordinates of the centre $= \left(\frac{0+6}{2}, \frac{2+0}{2}\right) = (3,1)$

The equation of the circle is

$(x-3)^2 + (y-1)^2 = 3^2 + 1^2$

$x^2 + y^2 - 6x - 2y = 0$

12. A

Let $x^2 + y^2 + Dx + Ey + F = 0$ be the equation of the circle, where D, E and F are constants.

$$\begin{cases} 26 - 5D + E + F = 0 & \ldots(1) \\ 29 + 2D + 5E + F = 0 & \ldots(2) \\ 98 + 7D - 7E + F = 0 & \ldots(3) \end{cases}$$

$(2)-(1)$:

$3 + 7D + 4E = 0 \quad \ldots(4)$

$(3)-(2)$:

$69 + 5D - 12E = 0 \quad \ldots(5)$

$(4) \times 3 + (5)$:

$78 + 26D = 0$

$D = -3$

Putting $D = -3$ into (4), we have $E = \frac{9}{2}$.

Putting $D = -3$ and $E = \frac{9}{2}$ into (1), we have $F = -\frac{91}{2}$.

Therefore, the equation of the circle is

$x^2 + y^2 - 3x + \frac{9}{2}y - \frac{91}{2} = 0$

$2x^2 + 2y^2 - 6x + 9y - 91 = 0$

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Part 3A – Intersection of straight line and circle

1. A   2. A   3. B   4. A

1. A

$$\begin{cases} x^2+y^2+2x-6y-k=0 \\ x=2-2y \end{cases}$$

Putting $x=2-2y$ in $x^2+y^2+2x-6y-k=0$, we have

$(2-2y)^2+y^2+2(2-2y)-6y-k=0$

$5y^2-18y+8-k=0$

Note that $\Delta>0$, we have

$(-18)^2-4(5)(8-k)>0$

$k>-\frac{41}{5}$

2. A

$$\begin{cases} x^2+y^2-10x+16y-2k-49=0 \\ y=\frac{4}{3}x-18 \end{cases}$$

Putting $y=\frac{4}{3}x-18$ in $x^2+y^2-10x+16y-2k-49=0$, we have

$$x^2+\left(\frac{4}{3}x-18\right)^2-10x+16\left(\frac{4}{3}x-18\right)-2k-49=0$$

$$\frac{25}{9}x^2-\frac{110}{3}x-2k-13=0$$

Note that $\Delta=0$, we have

$$\left(\frac{110}{3}\right)^2-4\left(\frac{25}{9}\right)(-2k-13)=0$$

$k=-67$

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3. B

The equation of L is $y = mx$.

$$\begin{cases} x^2 + y^2 + 6x - 2y + 5 = 0 \\ y = mx \end{cases}$$

Putting $y = mx$ in $x^2 + y^2 + 6x - 2y + 5 = 0$, we have

$x^2 + (mx)^2 + 6x - 2(mx) + 5 = 0$

$(1 + m^2)x^2 + (6 - 2m)x + 5 = 0$

Since L is a tangent to the circle, we have

$(6 - 2m)^2 - 4(1 + m^2)(5) = 0$

$2m^2 + 3m - 2 = 0$

$(m + 2)(2m - 1) = 0$

$m = -2$ or $m = \dfrac{1}{2}$

4. A

$$\begin{cases} 2x^2 + 2y^2 + cx + 6y - c = 0 \\ x = 2y - 1 \end{cases}$$

Putting $x = 2y - 1$ in $2x^2 + 2y^2 + cx + 6y - c = 0$, we have

$2(2y - 1)^2 + 2y^2 + c(2y - 1) + 6y - c = 0$

$5y^2 + (c - 1)y + 1 - c = 0$

Note that $\Delta < 0$, we have

$(c - 1)^2 - 4(5)(1 - c) < 0$

$(c + 19)(c - 1) < 0$

$-19 < c < 1$

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