COORDINATE GEOMETRY(II)

Form 6
Vol 5

Part 3B – Locus

1. Denote the locus of P by Γ.

   Note that Γ is the perpendicular bisector of HK.

   Slope of $\Gamma = (-1)\times\left(\frac{-8-7}{3-2}\right) = 15$

   The coordinates of the mid-point of $HK = \left(\frac{7-8}{2}, \frac{2+3}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$

   The equation of Γ is

   $y - \frac{5}{2} = 15\left(x + \frac{1}{2}\right)$

   $15x - y + 10$

2. Note that the locus of P is a circle with centre A and radius of 3 units.

   The equation of the locus of P is

   $(x-2)^2 + (y-4)^2 = 9$

   $x^2 + y^2 - 4x - 8y + 11 = 0$

3. Denote the required locus by Γ.

   Note that the equation of Γ is in the form of $y = 2x + c$, where c is a constant.

   The y-intercepts of the two given parallel lines are 5 and 9 respectively.

   So, we have $c = \frac{5+9}{2} = 7$.

   Thus, the equation of Γ is $y = 2x + 7$.

4. (a) Note that Φ is a circle with centre G and radius GA.

$GA=\sqrt{(7+23)^2+(-2-14)^2}=34$

The equation of Φ is

$(x+23)^2+(y-14)^2=1156$

$x^2+y^2+46x-28y-431=0$

(b) (i) Since B lies on Φ, we have

$(-39+23)^2+(k-14)^2=1156$

$k-14=-30$ or $k-14=30$

$k=-16$ or $k=44$ (rej.)

Note that Γ is the perpendicular bisector of AB and passes through G.

Slope of $\Gamma=(-1)\times\left(\dfrac{-39-7}{-16+2}\right)=-\dfrac{23}{7}$

The equation of Γ is

$y-14=-\dfrac{23}{7}(x+23)$

$23x+7y+431=0$

(ii) The distance between A and B

$=\sqrt{(-39-7)^2+(-16+2)^2}=34\sqrt{2}$

The distance between H and K

$=2\times34=68$

The area of the quadrilateral $AHBK$

$=\dfrac{68\times34\sqrt{2}}{2}$

$=1156\sqrt{2}$

5. (a) The slope of ST

$= \frac{17-2}{12+8}$

$= \frac{3}{4}$

The mid-point of TR

$= \left(\frac{12+6k^2}{2}, \frac{17-3-2k}{2}\right)$

$= (6+3k^2, 7-k)$

Note that Γ is parallel to L and ℓ.

So, the equation of Γ is

$y-(7-k) = \frac{3}{4}(x-6-3k^2)$

$3x-4y-9k^2-4k+10=0$

(b) (i) Note that either SGR or TGR is a straight line.

Case 1: TGR is a straight line.

Then TR is the diameter and thus $\angle RST = 90^\circ$.

slope of $RS = \frac{2+3+2k}{-8-6k^2} = -1 \div (\text{slope of } ST)$

$\frac{2k+5}{6k^2+8} = \frac{4}{3}$

$24k^2-6k+17=0$

$\Delta = 36-4(24)(17) = -1596 < 0$

$\therefore$ no solution of k

Case 2: SGR is a straight line.

i.e. SR is a diameter of C.

Thus, we have $ST \perp TR$.

$\frac{17+3+2k}{12-6k^2} = -\frac{4}{3}$

$4k^2-k-18=0$

$(4k-9)(k+2)=0$

$k = \frac{9}{4}$ (rej.) or $k=-2$

(ii) Let h be the perpendicular distance from P to L.

The area of ΔGTK

$$=\frac{h}{2}(GT)$$

The area of ΔQTH

$=$ The area of ΔQGH $+$ The area of ΔGTH

$$=\frac{h}{2}(GH)+\frac{h}{2}(GH)$$

$$=h(GH)$$

So, the required ratio is $2GH:GT$.

Note that GH and GT are radii of C.

Therefore, the required ratio is $2:1$.