COORDINATE GEOMETRY(II)

Form 6

Vol 5

Part 2 – Transformation (Trigo)

1.	A	2.	B	3.	C	4.	D	5.	C
6.	B	7.	D						

1. A

Note that the graph cuts the y-axis at $x = 0°, 180°$ and $360°$.

So, the function y is in the form of $a\sin x$ for some constant a.

Since $a\sin 90° = -2$, we have $a = -2$.

Thus, we have $y = -2\sin x$.

2. B

Note that the maximum of $y = a\cos(x° - \theta)$ is attained at $x = 210$.

If $a > 0$, the function $y = a\cos(x° - \theta)$ attains its maximum at $x° - \theta = 0°$ or $360°$.

However, it is impossible that $210° - \theta = 0°$ and $210° - \theta = 360°$ for $-90° < \theta < 90°$.

So, we have $a < 0$.

Since the maximum of y is 3, we have $a = -3$.

Note that the function $y = a\cos(x° - \theta)$ attains its maximum at $x° - \theta = 180°$.

$\therefore\ 210° - \theta = 180°$

$\theta = 30°$

3. C

If $a > 0$, the function $y = a\sin(x° + \theta)$ attains its minimum at $x° - \theta = 270°$.

However, the figure shows that $y = a\sin(x° + \theta)$ attains its minimum at $x° - \theta < 240°$.

So, we have $a < 0$.

Since the minimum of y is -3, we have $a = -3$.

Note that the graph of $y = a\sin(x° + \theta)$ cuts the x-axis at $x° + \theta = 0°$, $180°$ or $360°$.

So, for $x = 240$, we have

$240° + \theta = 0°$ or $240° + \theta = 180°$ or $240° + \theta = 360°$

$\theta = -240°$ or $\theta = -60°$ or $\theta = -120°$

Since $-90° < \theta < 90°$, we have $\theta = -60°$.

4. D

Consider a cosine function $\cos(\varphi)$ for $0 \le \varphi \le 360°$, we have $\cos(\varphi) = 0$ at $\varphi = 90°$ or $\varphi = 270°$.

Note that $\cos(kx° + \theta) = 0$ at $x = 10$ and $x = 70$.

So, we have $\begin{cases} k(10)° + \theta = 90° \\ k(70)° + \theta = 270° \end{cases}$.

Solving, we have $k = 3$ and $\theta = 60°$.

5. C

Note that the graph of $y = g(x)$ has a smaller period, i.e. the graph is compressed horizontally.

So, we have $a > 1$.

Also note that the maximum of $y = f(x)$ is 4.

$\therefore\ 2(4) + k = 0$.

$k = -8$

6. B

Note that the function is in the form of $a\sin bx + k$ where a, b and k are constants.

Since the amplitude of $f(x)$ remains 1, we have $a = 1$.

Since the minimum of $f(x)$ is -1, we have $k = -1$

However, the period of $f(x)$ is $180°$, so $b = 2$.

Thus, we have $f(x) = \sin 2x - 1$.

7. D

Let $y = f(x) = k + a\cos(bx)$, where a, b and k are constants.

Since the amplitude of $f(x)$ is 3, we have $a = 3$.

Note that the maximum of $f(x)$ is 7.

$\therefore\ 3(1) + k = 7$

$k = 4$

Also, the period of $f(x)$ is $180°$. So, $b = 2$.

Thus, we have $y = 4 + 3\cos(2x°)$.

Part 3A – Locus

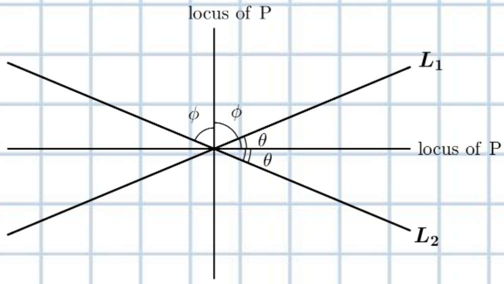
1.	D	2.	D	3.	B	4.	C	5.	C
6.	A	7.	C						

1. D

Note that the locus of P is a pair of angle bisectors.

With the notations in the figure, we can see that $\theta+\phi=90^\circ$.

So, the locus of P is a pair of perpendicular lines.



2. D

Note that the locus of C is the perpendicular bisector of AD.

In particular, the locus of C is perpendicular to AD.

So, the locus of C is parallel to BD.

3. B

II is not true:

Note that Γ is parallel to L_1 and passes through the mid-point of AB.

However, slope of Γ × slope of $AB=\frac{3}{2}\times\left(-\frac{95}{50}\right)=-\frac{57}{20}\neq-1$.

So, Γ is not perpendicular to AB.

III is true:

Suppose that L_1 cuts the y-axis at the point C.

Note that Γ passes through the mid-point of BC.

The y-coordinate of the mid-point of $BC=\left(\frac{\frac{50}{8}-\frac{95}{12}}{2}\right)=-\frac{5}{6}$

So, Γ passes through the point $\left(0,-\frac{5}{6}\right)$.

4. C

For I, the locus of P is a closed line made up of two parts:

(1) a pair of line segments, with length equal to AB, that are parallel to AB with distance d;

(2) two semi-circles each of radius d.

For II, the locus of P is a circle of radius 5 units.

(Remark: The locus of P and C are concentric circles.)

For III, the locus of P is a circle with diameter AB.

5. C

The coordinates of A and B are $(7,0)$ and $(0,-5)$ respectively.

Let $P(x, y)$.

$AP = 2BP$

$\sqrt{(x-7)^2 + y^2} = 2\sqrt{x^2 + (y+5)^2}$

$x^2 - 14x + 49 + y^2 = 4x^2 + 4y^2 + 40y + 100$

$3x^2 + 3y^2 + 14x + 40y + 51 = 0$

6. A

Note that Γ is a pair of straight lines which are parallel to L.

Slope of $L = -\frac{1}{2}$

So, the equations of Γ are in the form of $y = -\frac{1}{2}x + c$, where c is a constant.

The vertical distance between Γ and $L = \frac{2\sqrt{5}}{\cos\left[90° - \tan^{-1}(2)\right]} = 5$

So, we have $c - 1 = 5$ or $1 - c = 5$.

$\therefore$ $c = 6$ or $c = -4$

Therefore, the equations of Γ are $y = -\frac{1}{2}x + 6$ and $y = -\frac{1}{2}x - 4$ respectively.

i.e. $x + 2y - 12 = 0$ and $x + 2y + 8 = 0$.

Thus, only I is true.

7. C

Denote the locus of P by Γ.

Note that Γ is a circle with diameter AB.

The coordinates of the centre of $\Gamma = \left(\frac{1+2}{2}, \frac{4-1}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$

The radius of $\Gamma = \frac{1}{2}\sqrt{(2-1)^2+(-1-4)^2} = \frac{\sqrt{26}}{2}$

The equation of Γ is

$$\left(x-\frac{3}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\frac{13}{2}$$

$$x^2+y^2-3x-3y-2=0$$

Alternatively,

Let $P(x, y)$.

Slope of PA × Slope of $PB = -1$

$$\frac{y-4}{x-1}\times\frac{y+1}{x-2}=-1$$

$$y^2-3y-4=-x^2+3x-2$$

$$x^2+y^2-3x-3y-2=0$$