超研數學 ASPIRE MATHEMATICS

SOLUTION

超研真心CHUR　但超研真心勁

REGULAR QUIZ 02

Form 6

Coordinate geometry (I)

Part A - MC (@3 marks)

1.	D	Let R and r be the radius of larger and smaller circle respectively. $\begin{cases} 2\pi R + 2\pi r = 16\pi \\ \pi R^2 + \pi r^2 = 34\pi \end{cases}$ Solving, we have $R = 5$ The area of the larger circle $= 25\pi \text{ cm}^2$
2.	A	Note that α and β are the roots of $3(5^x)^2 - 20(5^x) + 9 = 0$. $(5^\alpha)(5^\beta) = 3$ $5^{\alpha+\beta} = 3$ $\alpha + \beta = \dfrac{\log 3}{\log 5} = \log_5 3$
3.	D	$\begin{cases} y = (2m+1)x^2 + 2x + 5 \\ y = -2x + 17 \end{cases}$ $(2m+1)x^2 + 4x - 12 = 0$ $\Delta > 0$ $4^2 - 4(2m+1)(-12) > 0$ $m > -\dfrac{2}{3}$
4.	B	$f(m-1)$ $= \left(2 \times \dfrac{m-1}{2} + 1\right)^2 - 4\left(\dfrac{m-1}{2}\right) + 5$ $= m^2 - 2m + 7$
5.	C	Since the graph passes through $(5,0)$ and has axis of symmetry $x = 2$ So, it must also pass through $(-1,0)$ $\therefore y = a(x+1)(x-5)$ Put $(2,6)$, then $a = -\dfrac{2}{3}$ $\therefore y = -\dfrac{2}{3}(x+1)(x-5)$

6.	D	$\begin{cases}(\alpha+2)+(\beta+2)=-3\\(\alpha+2)(\beta+2)=9\end{cases}$ $\therefore\begin{cases}(-\alpha)+(-\beta)=7\\(-\alpha)(-\beta)=19\end{cases}$ The required equation $x^2-7x+19=0$
7.	B	$\sqrt{x}-2\neq 0$ and $x\geq 0$ $x\neq 4$ $\therefore x$ can be all non-negative real number except 4.
8.	B	Since $ab>0$, then $-\dfrac{b}{2a}<0$ The x-coordinates of vertex < 0
9.	A	$y=5$ $\quad$ $y=17$ $5=x^2-6x+10$ $\quad$ $17=x^2-6x+10$ $x=1$ $\quad$ $x=7$ For $f(x)>g(x)$, then $1<x<7$
10.	B	$\dfrac{-6}{k}\times\dfrac{-2}{3}=-1$ $k=-4$
11.	D	slope of $L_1>0$, $\therefore b<0$ I is correct as y-int of $L_1>-1$, $-\dfrac{c}{b}>-1$, $\therefore b<c$ II is correct as x-int of $L_1=$ x-int of L_2, $-\dfrac{c}{4}=-\dfrac{m}{k}$, $\therefore ck-4m=0$ III is correct as slope of $L_1>$ slope of L_2, $-\dfrac{4}{b}>-k$, $\therefore bk<4$
12.	B	Since A translates a units leftwards and b units downwards to O So B also translates a units leftwards and b units downwards to C $\therefore C(c-a,d-b)$
13.	D	Let $y=mx+c$ is added. $mx+c=2x^2-15x+7$ $2x^2-(m+15)x+7-c=0$ $-x^2+\dfrac{m+15}{2}x+\dfrac{c-7}{2}=0$ $\dfrac{m+15}{2}=8$ $\quad$ $\dfrac{c-7}{2}=10$ $m=1$ $\quad$ $c=27$

14.	B	$\begin{cases} y = x^2 - 5x + 2 \\ y = mx + m \end{cases}$ $mx + m = x^2 - 5x + 2$ $x^2 - (m+5)x + 2 - m = 0$ x-coordinates of the mid-point of A and C $= \dfrac{m+5}{2}$
15.	C	x-coordinates of the vertex $= -\dfrac{k}{2(-3)} = \dfrac{k}{6}$ Put $\left(\dfrac{k}{6}, 22\right)$, then $k^2 = 324$ $AB = \dfrac{\sqrt{k^2 - 4(-3)(-5)}}{3}$ $= \dfrac{2\sqrt{66}}{3}$
16.	D	Note that $AE : EB = 1:1 = 4:4$. Since $\Delta BEH \sim \Delta CFH$, $EH : HF = 2:3 = 6:9$. Since $\Delta BEI \sim \Delta GFI$, $EI : IF = 4:11$. Therefore, $EI : IH : HF = 4:2:9$. Area of $\Delta EBH = 54 \times \left(\dfrac{4}{6}\right)^2 = 24 \text{ cm}^2$ Area of $\Delta EBI = 24 \times \dfrac{4}{6} = 16 \text{ cm}^2$ Area of $\Delta BIH = 24 \times \dfrac{2}{6} = 8 \text{ cm}^2$ Area of $\Delta IGF = 16 \times \left(\dfrac{11}{4}\right)^2 = 121 \text{cm}^2$ Area of $CHIG = 121 - 54 = 67 \text{ cm}^2$ $\dfrac{\text{Area of } AEBGD}{\text{Area of } \Delta BCG} = \dfrac{8+3}{5}$ $\dfrac{\text{Area of } ADGIE + 16}{67 + 8} = \dfrac{11}{5}$ Area of $ADGIE = 149 \text{ cm}^2$

1.	D	2.	A	3.	D	4.	B	5.	C
6.	D	7.	B	8.	B	9.	A	10.	B
11.	D	12.	B	13.	D	14.	B	15.	C
16.	D								

Part B - Long Questions (20 marks)

1. (10 marks)

(a) Put $y=0$, $x^2+2kx+5k-7=0$.

Δ

$=(2k)^2-4(5k-7)$ 1M

$=4k^2-20k+28$

$=4(k^2-5k)+28$

$=4\left[k^2-5k+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2\right]+28$ 1M

$=4(k-\frac{5}{2})^2+3$

≥ 3

>0

Thus, the graph always has 2 distinct x-intercepts. 1A f.t.

(b)(i) $AB=\sqrt{4(k-\frac{5}{2})^2+3}$ 1M

By (a), AB is the shortest when $k=\frac{5}{2}$.

Put $k=\frac{5}{2}$, $OC=\frac{11}{2}$ 1M

The area of ΔABC

$=\frac{AB(OC)}{2}$ 1M

$=\frac{11\sqrt{3}}{4}$ sq. units 1A

(b)(ii) Area of ΔABC

$=\frac{1}{2}(AB)(OC)$

$=\frac{1}{2}(5k-7)\sqrt{4\left(k-\frac{5}{2}\right)^2+3}$ 1M

When $k=\frac{7}{5}$, area of ΔABC is 0, which attains minimum. 1M

Thus, the claim is not agreed. 1A f.t.

2. (10 marks)

(a) x-intercept $= 4$, y-intercept $= -4$ 2A

(b) $L_1 : y = -x + 4$, $L_2 : y = 3x - 4$ 2A

(c) $\begin{cases} y = -x + 4 \\ y = 3x - 4 \end{cases}$ 1M

Hence, $D(2, 2)$ 1A

(d) Area of ΔBCD

$= \dfrac{(4+4)4}{2} - \dfrac{(4+4)2}{2} = 8$ sq. units 1M+1A

(e) Note that $DB \perp CB$ 1M

$\therefore K(4, 0)$ 1A