EQUATION OF CIRCLE

Form 5

Vol 6

Part 4 – Chord

1. $\begin{cases} y = \dfrac{x-k}{2} \\ x^2 + y^2 - 7x + 4y - 30 = 0 \end{cases}$

Putting $y = \dfrac{x-k}{2}$ into $x^2 + y^2 - 7x + 4y - 30 = 0$, we have

$$x^2 + \left(\frac{x-k}{2}\right)^2 - 7x + 4\left(\frac{x-k}{2}\right) - 30 = 0$$

$$\frac{5}{4}x^2 - \left(\frac{k}{2} + 5\right)x + \frac{k^2}{4} - 2k - 30 = 0$$

Thus, the x-coordinate of the mid-point of the chord $PQ = \dfrac{\frac{k}{2} + 5}{2\left(\frac{5}{4}\right)} = \dfrac{k+10}{5}$.

2. $\begin{cases} y = 3x - 40 \\ x^2 + y^2 - 18x - ky + 50 = 0 \end{cases}$

Putting $y = 3x - 40$ in $x^2 + y^2 - 18x - ky + 50 = 0$, we have

$$x^2 + (3x-40)^2 - 18x - k(3x-40) + 50 = 0$$

$$10x^2 - (3k + 258)x + 40k + 1650 = 0$$

So, the x-coordinate of the mid-point of the chord $AB = \dfrac{3k+258}{2(10)} = \dfrac{3k+258}{20}$

and the y-coordinate of the mid-point of the chord $AB = 3\left(\dfrac{3k+258}{20}\right) - 40 = \dfrac{9k-26}{20}$.

Thus, the coordinates of the mid-point of the chord AB are $\left(\dfrac{3k+258}{20}, \dfrac{9k-26}{20}\right)$.

3. The coordinates of the centre of the circle are $(4, 2)$.

Note that the line joining centre to the mid-point of PQ is perpendicular to PQ.

Slope of $PQ = (-1)\times\left(\frac{0-4}{3-2}\right) = 4$

Thus, the equation of L is $y = 4x+3$.

4. (a) Let the coordinates of the centre of the circle be $(2k-8, k)$.

$$(2k-8-8)^2 + (k-0)^2 = (2k-8-5)^2 + (k+3)^2$$

$$-64k + 256 = -52k + 169 + 6k + 9$$

$$k = \frac{13}{3}$$

Thus, the coordinates of the centre of the circle are $\left(\frac{2}{3}, \frac{13}{3}\right)$.

(b) The radius of the circle $= \sqrt{\left[2\left(\frac{13}{3}\right)-16\right]^2 + \left(\frac{13}{3}\right)^2} = \frac{\sqrt{653}}{3}$.

The equation of the circle is

$$\left(x-\frac{2}{3}\right)^2 + \left(y-\frac{13}{3}\right)^2 = \frac{653}{9}$$

$$x^2 + y^2 - \frac{4}{3}x - \frac{26}{3}y - \frac{160}{3} = 0$$

$$3x^2 + 3y^2 - 4x - 26y - 160 = 0$$

5. (a) The coordinates of the centre of the circle $= \left(\frac{\frac{32}{2}}{2}, -\frac{\frac{48}{2}}{2}\right) = (8, -12)$.

The radius of the circle $= \sqrt{(8)^2 + (-12)^2 - \left(\frac{336}{2}\right)} = 2\sqrt{10}$.

So, we have

$$(6-8)^2 + (k+12)^2 = (2\sqrt{10})^2 - \left(\frac{4\sqrt{5}}{2}\right)^2$$

$$(k+12)^2 = 16$$

$k = -16$ or $k = -8$

(b) Note that the line joining the centre to the mid-point of AB is perpendicular to AB.

For $k=-16$,

Slope of $AB=(-1)\times\left(\frac{6-8}{-16+12}\right)=-\frac{1}{2}$

The equation of L is

$y+16=-\frac{1}{2}(x-6)$

$x+2y+26=0$

For $k=-8$,

Slope of $AB=(-1)\times\left(\frac{6-8}{-8+12}\right)=\frac{1}{2}$

The equation of L is

$y+8=\frac{1}{2}(x-6)$

$x-2y-22=0$

6. (a) The coordinates of the centre of the circle $=\left(\frac{\frac{32}{2}}{2},-\frac{\frac{k}{2}}{2}\right)=\left(8,-\frac{k}{4}\right)$.

The radius of the circle $=\sqrt{(8)^2+\left(-\frac{k}{4}\right)^2-\left(\frac{308}{2}\right)}=\sqrt{\frac{k^2}{16}-90}$.

So, we have

$(10-8)^2+\left(12+\frac{k}{4}\right)^2=\frac{k^2}{16}-90-\left(\frac{4\sqrt{10}}{2}\right)^2$

$6k+148=-130$

$k=-\frac{139}{3}$

(b) Note that the line joining the centre to the mid-point of PQ is perpendicular to PQ.

Slope of $PQ=(-1)\times\left(\frac{10-8}{12-\frac{139}{12}}\right)=-\frac{24}{5}$

The equation of L is

$y-12=-\frac{24}{5}(x-10)$

$24x+5y-300=0$

7. Let the coordinates of the centre of C be $\left(h, \frac{3h+29}{4}\right)$.

$$(h-0)^2 + \left(\frac{3h+29}{4} - 9\right)^2 = (h-0)^2 + \left(\frac{3h+29}{4} - 1\right)^2$$

$$-\frac{9}{2}(3h+29) + 81 = -\frac{1}{2}(3h+29) + 1$$

$h = -3$

So, the coordinates of the centre of C are $(-3, 5)$.

The radius of C $= \sqrt{(-3)^2 + (5-1)^2} = 5$.

The equation of C is

$$(x+3)^2 + (y-5)^2 = 25$$

$$x^2 + y^2 + 6x - 10y + 9 = 0$$

8. Slope of $OR = (-1) \times \left(\frac{3}{4}\right) = -\frac{3}{4}$.

Let the coordinates of R be $\left(h, -\frac{3}{4}h\right)$.

$$4h - 3\left(-\frac{3}{4}h\right) - 50 = 0$$

$h = 8$

So, we have $R(8, -6)$.

The radius of $C = \sqrt{(8-0)^2 + (6-0)^2} = 10$.

The equation of C is

$$(x-8)^2 + (y+6)^2 = 100$$

$$x^2 + y^2 - 16x + 12y = 0$$

9. B

Note that the mid-point of MN lies on the line $hx-3y=12$.

So, we have $h(6)-3(4)=12$.

$\therefore\ h=4$

Slope of $MN=\frac{4}{3}$

The coordinates of the centre of the circle are $\left(7,-\frac{k}{4}\right)$.

Note that the line joining the centre to the mid-point of MN is perpendicular to MN.

$$\frac{-\frac{k}{4}-4}{7-6}=-\frac{3}{4}$$

$k=-13$

10. A

Slope of $PQ=\frac{3}{4}$

Note that the line joining the centre to the mid-point of PQ is perpendicular to PQ.

Let the coordinates of the mid-point of PQ be $\left(a,\frac{3a+20}{4}\right)$.

So, we have

$$\frac{\frac{3a+20}{4}-25}{a+15}=-\frac{4}{3}$$

$a=0$

So, the coordinates of the mid-point of PQ are $(0,5)$.

The perpendicular distance between the centre and the mid-point of PQ

$$=\sqrt{(0+15)^2+(5-25)^2}=25$$

The radius of $C=\sqrt{25^2+\left(\frac{50}{2}\right)^2}=25\sqrt{2}$

The equation of C is

$$(x+15)^2+(y-25)^2=1250$$

$$x^2+y^2+30x-50y-400=0$$

Part 5A – Tangent

1. C

Putting $x=-1,\ y=6$ in $x^2+y^2-(a+2)x-5ay+19=0$, we have

$(-1)^2+(6)^2-(a+2)(-1)-5a(6)+19=0$

$a=2$

So, the coordinates of the centre of the circle are $(2,5)$.

Slope of the tangent to the circle at $P=(-1)\times\left(\frac{2+1}{5-6}\right)=3$.

The equation of the tangent to the circle at P is

$y-6=3(x+1)$

$3x-y+9=0$

2. D

The coordinates of the centre of the circle are $(-4,-9)$.

Slope of the tangent to the circle at $(2,-1)=(-1)\times\left(\frac{-4-2}{-9+1}\right)=-\frac{3}{4}$.

The equation of the tangent to the circle at $(2,-1)$ is

$y+1=-\frac{3}{4}(x-2)$

$3x+4y-2=0$

3. C

The coordinates of the centre of the circle are $(-2,3)$.

Slope of the tangent to the circle at $(3,-9)=(-1)\times\left(\frac{-2-3}{3+9}\right)=\frac{5}{12}$.

The equation of the tangent to the circle at $(3,-9)$ is

$y+9=\frac{5}{12}(x-3)$

$5x-12y-123=0$

4. Since L_1 passes through P, we have

$3(-4)-k(2)+28=0$
$k=8$

Since C passes through P, we have

$(-4)^2+(2)^2+h(-4)-12(2)+48=0$

$h=11$

So, the coordinates of the centre of C are $\left(-\frac{11}{2},6\right)$.

Suppose that L_2 touches C at the point Q.

Then, the coordinates of $Q=\left(-\frac{11}{2}\times 2+4, 6\times 2-2\right)=(-7,10)$

Since $L_2 // L_1$, the slope of L_2 is $\frac{3}{8}$.

The equation of L_2 is

$y-10=\frac{3}{8}(x+7)$
$3x-8y+101=0$