EQUATION OF CIRCLE

Form 5

Vol 6

Part 2B – Condition of equation of circle

1. (a) The coordinates of the centre of C are $(2,3)$.

   (b) (i) The radius of C is 3.

   (ii) Note that the circle touches the *x*-axis at the point $(2,0)$.

   Putting $x=2,\ y=0$ into the equation of C, we have

   $$2^2-4(2)+k=0$$

   $$k=4$$

2. Note that the radius of C is 4.

   The equation of C is

   $$(x+4)^2+(y-3)^2=16$$

   $$x^2+y^2+8x-6y+9=0$$

3. Let the coordinates of the centre of the circle be $(6,k)$.

   $$k^2=(6-0)^2+(k-18)^2$$

   $$k=10$$

   So, the radius of the circle is 10.

   The equation of the circle is

   $$(x-6)^2+(y-10)^2=100$$

   $$x^2+y^2-12x-20y+36=0$$

4. Let the coordinates of the centre of C be $(h,0)$.

   $$h^2=(h+2)^2+(0-6)^2$$

   $$h=-10$$

   So, the radius of C is 10.

   The equation of C is

   $$(x+10)^2+y^2=100$$

   $$x^2+y^2+20x=0$$

5. Note that the centre of the inscribed circle is the incentre of ΔOAB.

Let $I(-r,r)$ be the centre of the inscribed circle.

$$\tan \angle ABO = \frac{24}{32}$$

$$\tan 2\angle IBO = \frac{3}{4}$$

$$\angle IBO = \frac{1}{2}\tan^{-1}\left(\frac{3}{4}\right)$$

So, we have $\tan \angle IBO = \frac{1}{3}$.

$$\frac{r}{-r+32} = \frac{1}{3}$$

$$r = 8$$

So, we have $I(-8,8)$ and the radius of the inscribed circle is 8.

The equation of the inscribed circle is

$$(x+8)^2 + (y-8)^2 = 64$$

$$x^2 + y^2 + 16x - 16y + 64 = 0$$

Alternatively,

Let p be the perimeter of ΔOAB,

r be the radius of the inscribed circle, and

a be the area of ΔOAB.

By $pr = 2a$, we have

$$(32 + 24 + \sqrt{32^2 + 24^2})r = 2\left[\frac{(32)(24)}{2}\right]$$

$$r = 8$$

So, the coordinates of the centre of the inscribed circle are $(-8,8)$.

The equation of the inscribed circle is

$$(x+8)^2 + (y-8)^2 = 64$$

$$x^2 + y^2 + 16x - 16y + 64 = 0$$

6. Let the coordinates of the centre of the circle be $(-r, r)$.

Note that $\tan \angle ABO = \frac{2}{3}$.

$$\frac{r}{-r+15} = \frac{2}{3}$$

$$r = 6$$

So, the coordinates of the centre of the circle are $(-6, 6)$ and the radius of the circle is 6.

The equation of the circle is

$$(x+6)^2 + (y-6)^2 = 36$$

$$x^2 + y^2 + 12x - 12y + 36 = 0$$

7. (a) Note that P, A and Q are collinear.

The radius of C_1 is 5.

The radius of C_2 is 5.

(b) $P(5,3)$

$Q(5,-7)$

(c) The equation of C_1 is

$$(x-5)^2 + (y-3)^2 = 25$$

$$x^2 + y^2 - 10x - 6y + 9 = 0$$

The equation of C_2 is

$$(x-5)^2 + (y+7)^2 = 25$$

$$x^2 + y^2 - 10x + 14y + 49 = 0$$

8. The radius of the circle

$= \sqrt{3^2 + \left(\frac{6}{2}\right)^2}$

$= 3\sqrt{2}$

The equation of the circle is

$(x+3)^2 + (y-5)^2 = 18$

$x^2 + y^2 + 6x - 10y + 16 = 0$

9. (a) The radius of the circle

$= \sqrt{3^2 + \left(\frac{8}{2}\right)^2}$

$= 5$

(b) The equation of the circle is

$(x-3)^2 + (y-3)^2 = 25$

$x^2 + y^2 - 6x - 6y - 7 = 0$

10. The radius of the circle

$= \sqrt{\left(\frac{10}{2}\right)^2 + 12^2}$

$= 13$

The equation of the circle is

$(x-7)^2 + (y-12)^2 = 169$

$x^2 + y^2 - 14x - 24y + 24 = 0$