REGULAR QUIZ 03

Form 5

Locus

1. (3 marks)

Let $P(x, y)$

$\frac{y-3}{x-1} \times \frac{y-5}{x-2} = -1$ 1M

$(y-3)(y-5) = -(x-1)(x-2)$

$y^2 - 8y + 15 = -x^2 + 3x - 2$

$x^2 + y^2 - 3x - 8y + 17 = 0$ 2A (accept not in general form)

2. (5 marks)

(a) Let $P(x, y)$

The equation of the locus

$(x-4)^2 + (y-1)^2 = (x-2)^2 + (y+3)^2$ 1M

$-8x + 16 - 2y + 1 = -4x + 4 + 6y + 9$

$x + 2y - 1 = 0$ 1A

(b) Note that $y = 4$ is perpendicular bisector of AC. 1M

Put $O(a, 4)$ into $x + 2y - 1 = 0$, $a = -7$ 1M

$O(-7, 4)$ 1A

3. (5 marks)

(a) Let $P(x, y)$

Note that $L_1 // L_2$ with slope $= \frac{4}{3}$ 1M

y-int of $L_1 = 8$

y-int of $L_2 = \frac{3}{2}$

The equation of the locus of P

$y = \frac{4}{3}x + \left(\frac{8+1.5}{2}\right)$ 1M

$y = \frac{4}{3}x + \frac{19}{4}$ 1A (or $16x - 12y + 57 = 0$)

(b) Note that $AC : AB = 1:2$ 1M

Height of $\Delta ACQ =$ Height of ΔABQ

area of ΔACQ : area of $\Delta ABQ = 1:2$ 1A

4. (7 marks)

(a) The equation of L

$\frac{y-1}{x-0} = \frac{0-1}{2-0}$ 1M

$x + 2y - 2 = 0$ 1A (or $y = -\frac{1}{2}x + 1$)

(b) Let $P(x, y)$

The equation of locus

$\frac{x+2y-2}{\sqrt{1^2+2^2}} = 2\sqrt{5}$ or $\frac{x+2y-2}{\sqrt{1^2+2^2}} = -2\sqrt{5}$ 2M+2A

$x + 2y - 12 = 0$ $x + 2y + 8 = 0$

Alternative solution

Note that slope of $L=-\frac{1}{2}$

Let $\tan\theta=\frac{1}{2}$

$c=\frac{2\sqrt{5}}{\cos\theta}=5$ 1M

The equation of locus

$y=-\frac{1}{2}x+1+5$ or $y=-\frac{1}{2}x+1-5$ 1M

$y=-\frac{1}{2}x+6$ or $y=-\frac{1}{2}x+-4$ 2A

$(x+2y-12=0$ or $x+2y+8=0)$

(c) A pair of straight lines parallel to L. 1A

5. (13 marks)

(a)(i) Let $P(x,y)$

The equation of Γ

$(x-1)^2+(y-5)^2=(x+1)^2+(y-3)^2$ 1M

$-2x+1-10y+25=2x+1-6y+9$

$x+y-4=0$ 1A

Γ is the perpendicular bisector of AB. 1A

(a)(ii) $M(0,4)$ 1A

(b)(i) Note that $L: y=4$ 1M

The equation of Φ

$(x-2)^2+(y-3)^2=(4-y)^2$ 1M

$x^2-4x+4-6y+9=16-8y$

$y=-\frac{1}{2}(x^2-4x-3)$ 1A (or equivant) (b)(ii) $\begin{cases} x+y-4=0 \\ y=-\dfrac{1}{2}(x^2-4x-3) \end{cases}$

$4-x=-\dfrac{1}{2}(x^2-4x-3)$ 1M

$x^2-6x+5=0$

$x=1$ or $x=5$

$y=3$ or $y=-1$

$R(1,3)$, $S(5,-1)$ 2A (or $R(5,-1)$, $S(1,3)$)

(b)(iii) For $R(1,3)$, $S(5,-1)$

height of ΔARM = height of ΔASM 1M

area of ΔARM : area of ΔASM

$= RM : SM$ 1M

$=\sqrt{(1-0)^2+(3-4)^2}:\sqrt{(5-0)^2+(-1-4)^2}$

$=1:5$ 1A

Alternative solution

For $R(5,-1)$, $S(1,3)$

height of ΔARM = height of ΔASM 1M

area of ΔARM : area of ΔASM

$= RM : SM$ 1M

$=\sqrt{(5-0)^2+(-1-4)^2}:\sqrt{(1-0)^2+(3-4)^2}$

$=5:1$ 1A

Bonus Question

(3 marks)

(a) Area of ΔABC

$=\dfrac{2(7-3)}{2}$

$=4$ 1A

(b) The equation of the locus of P

$\dfrac{y-7}{x-0}=\dfrac{7-3}{2-4}$ 1M

$2x+y-7=0$ 1A