COORDINATE GEOMETRY(I)

Form 6
Vol 4

Part 8 – Vertex/Intersection

1. (a) The y-intercept is -1.

(b) $2x^2+4x-1=0$

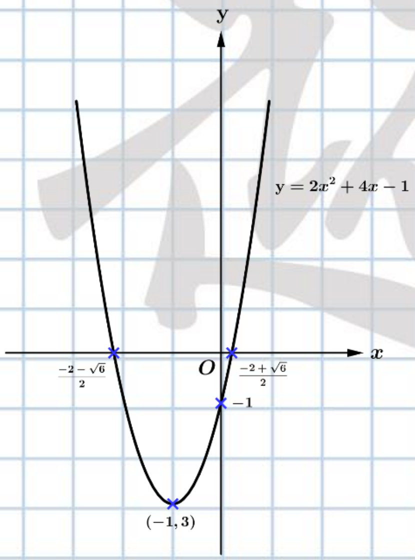
$x=\dfrac{-2\pm\sqrt{6}}{2}$

The x-intercepts are $\dfrac{-2+\sqrt{6}}{2}$ and $\dfrac{-2-\sqrt{6}}{2}$.

(c) $f(x)=2(x+1)^2-3$

The coordinates of the vertex is $(-1,-3)$.

(d)



2. B

3. The coordinates of V is $(2,9)$.

Putting $x=0$ into $y=9-(x-2)^2$, we have $y=5$.

Thus, we have $C(0,5)$.

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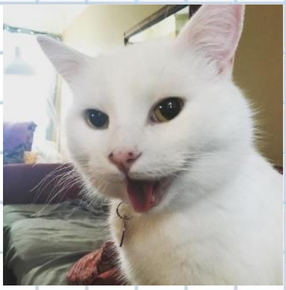

$$(x-2)^2=9$$
$$x-2=\pm 3$$
$$x=-1 \text{ (rej.) or } x=5$$

Thus, we have $B(5,0)$.

The area of OCVB

$$=\frac{1}{2}(5+9)(2)+\frac{1}{2}(9)(3)$$
$$=27.5$$

4. $3x^2+7x-1=0$

$$2x^2=-\frac{14}{3}x+\frac{2}{3}$$
$$2x^2-5x=-\frac{29}{3}x+\frac{2}{3}$$
$$2x^2-5x+1=-\frac{29}{3}x+\frac{5}{3}$$

Therefore, $y=-\frac{29}{3}x+\frac{5}{3}$ should be added.

5. $x^2+\frac{3}{2}x-4=0$

$$-x^2+1=\frac{3}{2}x-3$$

Therefore, $y=\frac{3}{2}x-3$ should be added.

6. (a) $\begin{cases} y=x^2-x-3 \\ y=3x+k \end{cases}$

$$x^2-4x-(3+k)=0$$
$$a_1+b_1=4$$
$$a_1b_1=-3-k$$

(b) The x-coordinate of the mid-point $=\frac{a_1+b_1}{2}=2$

Put $x=2$ into $y=3x+k$,

$$y=3(2)+k$$
$$=6+k$$

Thus, the coordinates of the mid-point of A and B is $(2,k+6)$.

(c) Since $a_1 < 0 < b_1$, P is between A and B.

By section formula,

$$P = \left(\frac{3a_1 + 2b_1}{5}, \frac{3a_2 + 2b_2}{5}\right)$$

Since P is the y-intercept of the line, we have $P(0, k)$.

$$\frac{3a_1 + 2b_1}{5} = 0 \quad \text{and} \quad \frac{3a_2 + 2b_2}{5} = k$$

$$3a_1 + 2b_1 = 0 \quad \text{and} \quad \frac{3a_2 + 2b_2}{5} = k$$

From (a), $a_1 + b_1 = 4$.

$$\begin{cases} a_1 + b_1 = 4 \\ 3a_1 + 2b_1 = 0 \end{cases}$$

By solving, we have $a_1 = -8$, $b_1 = 12$.

From (a), $a_1 b_1 = -3 - k$.

$$(-8)(12) = -3 - k$$
$$k = 93$$

7. B

Idea:
加 ***y* = ? (橫線)** 先可以同 **f(*x*)**相交 **3** 點

8. (a) $f(x) = 5x^2 + (2k - 13)x + (-5k - 5)$

$$\begin{aligned} \Delta &= (2k - 13)^2 - 4(5)(-5k - 5) \\ &= 4k^2 + 48k + 269 \\ &= 4(k^2 + 12k) + 269 \\ &= 4(k + 6)^2 + 125 \end{aligned}$$

Since $(k + 6)^2 \geq 0$, we have $\Delta > 0$.

Therefore, the graph of $y = f(x)$ cuts the x-axis at two distinct points.

(b) (i) The distance between A and B

$$= \frac{\sqrt{\Delta}}{5}$$

$$= \frac{\sqrt{4k^2 + 48k + 269}}{5}$$

So, we have

$$\frac{\sqrt{4k^2 + 48k + 269}}{5} = 3$$

$$4k^2 + 48k + 44 = 0$$

$$k^2 + 12k + 11 = 0$$

$(k+1)(k+11)=0$

$k=-1\text{(rej.)}$ or $k=-11$

(ii) (1) $y=f(x)$

$=5x^2-35+50$

$=5(x-2)(x-5)$

Thus, we have $A(2,0)$ and $B(5,0)$.

Note that the equation of the axis of symmetry is $x=\frac{7}{2}$.

When $x=\frac{7}{2}$, we have $f\left(\frac{7}{2}\right)=-\frac{45}{4}$.

So, the coordinates of C is $\left(\frac{7}{2},-\frac{45}{4}\right)$.

The coordinates of $H=\left(\frac{2+5+\frac{7}{2}}{3},\frac{-\frac{45}{4}}{3}\right)=\left(\frac{7}{2},-\frac{15}{4}\right)$.

Note that B, H and K are collinear.

Let the coordinates of K be $(a,5a^2-35a+50)$.

$$\frac{5a^2-35a+50}{a-5}=\frac{\frac{15}{4}}{5-\frac{7}{2}}$$

$$\frac{5a^2-35a+50}{a-5}=\frac{5}{2}$$

$$2a^2-15a+25=0$$

$$(2a-5)(a-5)=0$$

$$a=\frac{5}{2} \text{ or } a=5\text{(rej.)}$$

Thus, the coordinates of K is $\left(\frac{5}{2},-\frac{25}{4}\right)$.

(2) Let r be a constant such that $HB:BK=1:r$.

$$\frac{\frac{5}{2}+5r}{r+1}=\frac{7}{2}$$

$$r=\frac{2}{3}$$

$$\frac{HB}{BK}=\frac{1}{\frac{2}{3}}=\frac{3}{2}$$

Thus, we have $HB:BK=3:2$.

(3) Note that $DH : HC = 1:2$.

So, the ratio of the area of ΔDBH to the area of ΔBHC is $1:2$.

Also, note that the ratio of the area of ΔBHC to the area of ΔCHK is $3:2$.

Thus, the required ratio is $3:4$.

9. (a) Sub. $x=-2$, we have

$f(-2)$

$=\frac{1}{k-1}\left[(-2)^2-2k(-2)+8(-2)+10k-2\right]$

$=\frac{1}{k-1}(4+4k-16+10k-2)$

$=\frac{14k-14}{k-1}$

$=14$

Thus, the graph of $y=f(x)$ passes through N.

(b) (i) The x-coordinates of M

$=(-2)(10)=-20$

When $x=-20$, we have $f(-20)=\frac{50k+238}{k-1}$.

So, the coordinates of M is $\left(-20,\frac{50k+238}{k-1}\right)$.

$\frac{\left(\frac{50k+238}{k-1}\right)+(-6k)(9)}{10}=14$

$238+50k=(140+54k)(k-1)$

$54k^2+36k-378=0$

$3k^2+2k-21=0$

$(3k-7)(k+3)=0$

$k=\frac{7}{3}$(rej.) or $k=-3$

(ii) (1) Note that the y-coordinate of A is 18.

Then, we have $AB:NB=18:14=9:7$.

The coordinates of B

$=\left(\frac{(-2)(9)}{9-7},0\right)$

$=(-9,0)$

(2) $f(x) = -\frac{1}{4}(x^2 + 14x - 32)$

$= -\frac{1}{4}(x+7)^2 + \frac{7^2 + 32}{4}$

So, the equation of the axis of symmetry is $x = -7$.

The distance between H and K

$= \sqrt{(-14)^2 - 4(-32)}$

$= 18$

$BH = \frac{18}{2} - (-7 + 9) = 7$

$BK = 18 - 7 = 11$

Thus, we have $BH : BK = 7 : 11$.

(3) Since $AN : NB = 2 : 7$ and $AN : NM = 1 : 9$, we have

$NB : NM = 7 : 18$

$\Rightarrow NB : BM = 7 : 11$

So, the ratio of the area of ΔBMH to the area of ΔBHN is $11 : 7$.

Also note that the ratio of the area of ΔBHN to the area of ΔBNK is $7 : 11$.

Thus, the ratio of the area of ΔBMH to the area of ΔBNK is $1 : 1$.

i.e. Area of ΔBMH = Area of ΔBNK

The claim is agreed.