COORDINATE GEOMETRY(I)

Form 6
Vol 4

Part 7 – Quadratic graph

1. (a) Note that $\Delta < 0$.

$$3^2 - 4\left(\frac{1}{k}\right)(-4+3k) < 0$$

$$k > \frac{16}{3}$$

(b) (i) Note that the equation of the axis of symmetry is $x = -12$.

$$-\frac{3}{2\left(\frac{1}{k}\right)} = -12$$

$$k = 8$$

(ii) The y-coordinate of Q is $\frac{1}{8}p^2 + 3p + 20$.

The area of the rectangle $ORQS$

$$= (-p)\left(\frac{1}{8}p^2 + 3p + 20\right)$$

$$= -\frac{1}{8}p^3 - 3p^2 - 20p$$

(iii) Putting $x = 12$ into $y = \frac{1}{8}x^2 + 3x + 20$, we have $y = 2$.

Thus, the coordinates of V is $(-12, 2)$.

For $-12 < p < 0$, the area of $\Delta VPQ = \dfrac{-\frac{1}{8}p^3 - 3p^2 - 20p}{2} + \dfrac{(p+12)\left(\frac{1}{8}p^2 + 3p + 20\right)}{2} - \dfrac{(-p)(2)}{2}$

$$= 6\left(\frac{1}{8}p^2 + 3p + 20\right) + p$$

$$= \frac{3}{4}p^2 + 19p + 120$$

For $p < -12$, the area of $\Delta VPQ = \dfrac{(12)\left(\frac{1}{8}p^2 + 3p + 20\right)}{2} + \dfrac{(-p)\left(\frac{1}{8}p^2 + 3p + 20 - 2\right)}{2} - \dfrac{\left(-\frac{1}{8}p^3 - 3p^2 - 20p\right)}{2}$

$$= 6\left(\frac{1}{8}p^2 + 3p + 20\right) + p$$

$$= \frac{3}{4}p^2 + 19p + 120$$

Thus, the area of ΔVPQ is $\frac{3}{4}p^2+19p+120$ when P and V are distinct.

$$-\frac{1}{8}p^3-3p^2-20p=2\left(\frac{3}{4}p^2+19p+20\right)$$
$$p^3+24p^2+160p=-12p^2-304p-1920$$
$$p^3+36p^2+464p+1920=0$$

When $p=-8$, $(-8)^3+36(-8)^2+464(-8)+1920=0$.

i.e. $p=-8$ is a factor of $p^3+36p^2+464p+1920=0$.

So, we have $(p+8)(p^2+28p+240)=0$.

Consider $p^2+28p+240=0$,

$$\Delta=(28)^2-4(1)(240)$$
$$=-176<0$$

$\therefore$ No real solutions for $p^2+28p+240=0$.

So, $p=-8$ is the unique real solution for $p^3+36p^2+464p+1920=0$.

Putting $x=-8$, into $y=\frac{1}{8}x^2+3x+20$, we have $y=4$.

Therefore, $(-8,4)$ is the only position of P such that the area of the rectangle $OQPR$ is twice the area of ΔOPV.

2. (a) (i) $x=-\frac{3}{4}$

(ii) The distance between A and B

$=\sqrt{\left(-\frac{3}{2}\right)^2-4\left(-\frac{k}{2}\right)}$

$=\sqrt{\frac{9}{4}+2k}$

So, we have

$\sqrt{\frac{9}{4}+2k}=\frac{5}{2}$

$\frac{9}{4}+2k=\frac{25}{4}$

$k=2$

(b) (i) The distance between M and N

$=\frac{5}{2}\times\frac{7}{5}$

$=\frac{7}{2}$

Let the coordinates of N be n.

$2\left(n+\frac{3}{4}\right)=-\frac{7}{2}$

$n=1$

Putting $x=1$ into $y=-2x^2-3x+2$, we have $y=-3$.

Thus, the coordinates of N is $(1,-3)$.

(ii) The area of the trapezium $MNAB$

$=\frac{\left(\frac{5}{2}+\frac{7}{2}\right)(3)}{2}$

$=9$

3. (a) Note that $\Delta = 0$.

$$\left(\frac{k}{2}\right)^2 - 4\left(-\frac{1}{2}\right)\left(-\frac{21}{2}+k\right) = 0$$

$k^2 + 8k - 84 = 0$

$(k-6)(k+14) = 0$

$k = 6$ or $k = -14$ (rej.)

(b) Note that the equation of the axis of symmetry is $x = 3$ and the y-intercept is $-\frac{9}{2}$.

Thus, we have $A(3,0)$ and $B\left(0,-\frac{9}{2}\right)$.

(c) (i) The distance between A and R

$= \frac{9}{2} \times \frac{8}{9}$

$= 4$

Thus, the coordinates of R is $(7,0)$.

Let the coordinates of P be $(7, p)$.

Sub. $P(7, p)$ into $y = \frac{1}{2}(6x - x^2) - \frac{9}{2}$, we have

$$p = \frac{1}{2}\left[6(7) - (7)^2\right] - \frac{9}{2} = -8$$

Thus, the coordinates of P is $(7,-8)$.

(ii) The distance between P and Q

$= 2(7-3)$

$= 8$

The area of ΔBPQ

$= \frac{(8)(8-4.5)}{2}$

$= 14$

The area of ΔABP

$= \frac{(4.5+8)(7)}{2} - \frac{(4.5)(3)}{2} - \frac{(8)(4)}{2}$

$= 21$

Thus, the area of the quadrilateral $APQB = 14 + 21 = 35$.

4. (a) Note that the equation of the axis of symmetry is $x=\frac{3}{2}$.

$-\frac{k}{4}=\frac{3}{2}$

$k=-6$

(b) $y=2x^2-6x-36$

$=2(x^2-3x-18)$

$=2(x-6)(x+3)$

Thus, we have $A(6,0)$, $B(-3,0)$ and $C(0,-36)$.

(c) The area of ΔBMC

$=\frac{\left(\frac{3}{2}+3\right)(36)}{2}$

$=81$

Putting $x=\frac{3}{2}$ into $y=2x^2-6x-36$, we have $y=-\frac{81}{2}$.

Thus, the coordinates of V is $\left(\frac{3}{2},-\frac{81}{2}\right)$.

The area of ΔAVC

$=\frac{\left(36+\frac{81}{2}\right)\left(\frac{3}{2}\right)}{2}+\frac{\left(\frac{81}{2}\right)\left(6-\frac{3}{2}\right)}{2}-\frac{(36)(6)}{2}$

$=\frac{81}{2}$

Thus, the area of ΔBMC is twice the area of ΔAVC.

The claim is agreed.

5. C

Vertex $\left(\frac{3}{a}, b\right)$ which lies in quadrant IV.

$\frac{3}{a} > 0$

$\therefore a > 0$

$b < 0$

6. B

Vertex $(-b, c)$ which lies in quadrant IV.

$\therefore -b > 0$

$\therefore b < 0$

$\therefore c < 0$

$\because$ The graph open downwards,

$\therefore a < 0$

7. D

$\because$ The graph open upwards

$\therefore a > 0$

$\because$ By the method of completing square,

Vertex of $y = \left(\frac{-b}{2a}, c - \frac{b^2}{4a^2}\right)$

$\because$ Vertex lies in quardrant IV

$\therefore \frac{-b}{2a} > 0$

$\therefore -b > 0$

$\therefore b < 0$

$\because$ $y-$intercept is negative

$\therefore c < 0$

8. C

$\because$ The graph opens downwards

$\therefore a < 0$

$\because$ By the method of completing square,

Vertex of $y = \left(\frac{-b}{2a}, c - \frac{b^2}{4a^2}\right)$

$\because$ Vertex lies in quardrant II

$\therefore \frac{-b}{2a} < 0$

$\therefore -b > 0$

$\therefore b < 0$

$\because$ $y-$intercept is positive

$\therefore c > 0$

9. D

$\because$ The graph opens downwards

$\therefore a<0$

$\because$ By the method of completing square,

Vertex of $y=\left(\frac{-b}{2a},c-\frac{b^2}{4a^2}\right)$

$\because$ Vertex lies on the negative x-axis

$\therefore \frac{-b}{2a}<0$

$\therefore -b>0$

$\therefore b<0$

$\because$ $y-$intercept is negative

$\therefore c<0$

10. A

$\because a>0,$

$\therefore$ The graph opens upwards

$\because c>0,$

$\therefore$ The graph of y-intercept will also be positive

11. C

$\because$ Vertex of $y=(-b,c)$

$\because b<0$ and $c>0$

$\therefore -b>0$

$\therefore$ The vertex should be lies in quadrant I

12. D

$\because a<0$ and $b>0$

$\therefore$ Vertex of $y=(ab,a)$

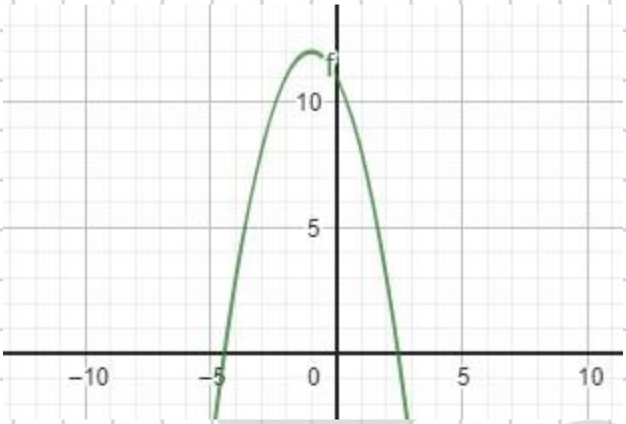
$\therefore$ Vertex lies in quadrant III

13. B

Statement I is true.

Statement II is true.

Statement III is false.



14. C

$\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}(k+h)x - \frac{1}{2}hk$

$\because \frac{-1}{2} < 0$

$\therefore$ The graph open downwards

$\because hk > 0$

$\therefore -\frac{1}{2}hk < 0$

$\therefore$ The y-intercept of the graph is negative

$\therefore x-$coordinate of the vertex $= \frac{-\frac{1}{2}(h+k)}{2\left(-\frac{1}{2}\right)}$

$\therefore x-$coordinate of the vertex is $\frac{h+k}{2}$