COORDINATE GEOMETRY(I)

Form 6
Vol 4

Part 3 – Solving quadratic equation

Reminder

開方記得加正負！

1. (a) $(3x+1)^2=(7x)^2$

$3x+1=7x$ or $3x+1=-7x$

$x=\frac{1}{4}$ or $x=-\frac{1}{10}$

(b) $y^2-2\sqrt{2}y+2=0$

$(y-\sqrt{2})^2=0$

$y=\sqrt{2}$

(c) $3x-2=1$ or $3x-2=-8$

$x=1$ or $x=-2$

(d) $(y-120)(y-1)=0$

$y=120$ or $y=1$

(e) $8x^2+14x-15=0$

$x=\frac{3}{4}$ or $x=-\frac{5}{2}$

2. $(x-a)(x-a-2)=(a-x)$

$(x-a)(x-a-2)=-(x-a)$

$(x-a)(x-a-2)+(x-a)=0$

$(x-a)(x-a-2+1)=0$

$(x-a)(x-a-1)=0$

$x=a$ or $x=a+1$

3. $(x+3c)(4c-x)=(2x+6c)(c-x)$

$(x+3c)(4c-x)=2(x+3c)(c-x)$

$(x+3c)(4c-x)-2(x+3c)(c-x)=0$

$(x+3c)(4c-x-2c+2x)=0$

$(x+3c)(x+2c)=0$

$x=-3c$ or $x=-2c$

4. $(2x+k)(30x-18k)=(x+k)(15x-9k)$

$6(2x+k)(5x-3k)=3(x+k)(5x-3k)$

$(4x+2k)(5x-3k)-(x+k)(5x-3k)=0$

$(4x+2k-x-k)(5x-3k)=0$

$(3x+k)(5x-3k)=0$

$x=-\frac{k}{3}$ or $x=\frac{3k}{5}$

Part 4 – Word problems

1. (a) $10(9-x)+x=90-9x$

(b) $90-9x=2(9-x)x$

$2x^2-27x+90=0$

$x=6$ or 7.5 (rej.)

$\therefore$ The two-digit number is 36.

2. Let x cm be the width of the cardboard.

Then, the length of the cardboard is $(x+8)$ cm.

$x(x+8)=105$

$x^2+8x-105=0$

$x=7$ or -15(rej.)

$\therefore$ The width is 7 cm and the length is 15 cm.

3. $\frac{(2x+5x-3)(x+1)}{2}=36$

$7x^2+4x-75=0$

$x=3$ or $x=-\frac{25}{7}$(rej.)

4. (a) Base area $=(x-4)(x+4)=(x^2-16)$ cm^2

(b) $2(x^2-16)=168$

$x^2=100$

$x=10$ or $x=-10$(rej.)

The required perimeter

$=2[10+(10+8)]$

$=56$ cm

5. Let x be the length of the one part of the wire.

$$\left(\frac{x}{4}\right)^2+\left(\frac{140-x}{4}\right)^2=725$$

$$x^2+19600-280x+x^2=11600$$

$$2x^2-280x+8000=0$$

$$x=100 \text{ or } 40$$

$\therefore$ The areas of the two squares are 100 cm^2 and 625 cm^2 respectively.

6. (a) The required distance $=\sqrt{13^2-12^2}=5\text{ m}$

(b) $13^2-(12-x)^2=(5+x)^2$

$$169-144+24x-x^2=25+10x+x^2$$

$$2x^2-14x=0$$

$$x=7 \text{ or } x=0 \text{ (rej.)}$$

7. (a) The required height $=\sqrt{10^2-6^2}=8\text{ m}$

(b) $10^2-(8-x)^2=(6+x)^2$

$$2x^2-4x=0$$

$$x=2 \text{ or } 0 \text{ (rej.)}$$

8. Let x km/h be the original speed.

$$\frac{15}{x}-\frac{15}{x+5}=\frac{5}{60}$$

$$\frac{15x+75-15x}{x^2+5x}=\frac{1}{12}$$

$$x^2+5x-900=0$$

$$x=\frac{-5+5\sqrt{145}}{2} \text{ or } \frac{-5-5\sqrt{145}}{2} \text{ (rej.)}$$

The original speed

$$=\frac{-5+5\sqrt{145}}{2}$$

$$\approx 27.6 \text{ km/h}$$

Part 5A – Delta/α/β

1. $12^2 - 4(3k)(4) < 0$

$k > 3$

2. $(4-k)x^2 + 24x + 9 = 0$

$\Delta < 0$

$24^2 - 4(4-k)(9) < 0$

$576 - 144 + 36k < 0$

$k < -12$

3. (a) $4x^2 - 8x + 3 - m = 0$

$\Delta \geq 0$

$8^2 - 4(4)(3-m) \geq 0$

$64 - 48 + 16m \geq 0$

$16m \geq -16$

$m \geq -1$

(b) Take $m = -1$

$4x^2 - 8x + 4 = 0$

$(x-1)^2 = 0$

$x = 1$

4. $\Delta = (k+4)^2 - 4(k+1)$

$= k^2 + 8k + 16 - 4k - 4$

$= k^2 + 4k + 12$

$= (k+2)^2 + 8$

≥ 8

> 0

Thus, the quadratic equation $x^2 - (k+4)x + k + 1 = 0$ always has real roots for all real values of k.