EQUATION OF CIRCLE

Form 5
Vol 6

Part 1 – General/Standard form

1.	A	2.	B	3.	B	4.	C	5.	C
6.	A	7.	B	8.	C	9.	D	10.	A
11.	B	12.	B	13.	B	14.	A	15.	D
16.	B	17.	B	18.	B				

1. A

III is not true:

Putting $x = 0$ into the equation of the circle, we have

$(y-3)^2 = 0$

Since $\Delta = 0$, the circle intersects the y-axis at 1 point only.

2. B

I is true:

The coordinates of the centre of the circle are $(-7,1)$,

which showing that the centre lies in the second quadrant.

II is not true:

Note that the radius of the circle is 6.

So, the diameter of the circle is 12.

III is true:

Note that $(0+7)^2 + (0-1)^2 = 50 > 36$.

The origin lies outside the circle.

3. B

I is not true:

The coordinates of the centre of the circle are $(0,5)$,

which showing that the centre lies on the y-axis.

II is true:

Note that $(-2)^2+(1-5)^2=20<25$.

So, the point $(-2,1)$ lies inside the circle.

III is not true:

The radius of the circle is 5.

The perimeter of the circle

$=10\pi$

≈ 31.41592654

>30

4. C

I is true:

Putting $x=-1$ into the equation of the circle, we have

$(-1)^2+y^2=1$

$y^2=0$

Since $\Delta=0$, the line $x+1=0$ touches the circle.

II is not true:

For any points $\left(x,\frac{3}{2}\right)$ on the line $2y-3=0$, we have

$x^2+\left(\frac{3}{2}\right)^2=x^2+\frac{9}{4}>1$,

which shows that the line $2y-3=0$ lies above the circle.

The line $2y-3=0$ does not intersect the circle.

III is true:

The coordinates of the centre of the circle are $(0,0)$.

It is clear that $(0,0)$ lies on the straight line $x+y=0$.

5. C

I is not true:

The coordinates of the centre of the circle $=\left(\dfrac{\frac{10}{2}}{2},-\dfrac{\frac{18}{2}}{2}\right)=\left(\dfrac{5}{2},-\dfrac{9}{2}\right)$.

II is true:

The radius of the circle $=\sqrt{\left(\dfrac{5}{2}\right)^2+\left(-\dfrac{9}{2}\right)^2-\left(\dfrac{-1}{2}\right)}=\sqrt{27}=3\sqrt{3}$.

III is true:

The distance between the origin and the centre

$=\sqrt{\left(\dfrac{5}{2}-0\right)^2+\left(-\dfrac{9}{2}-0\right)^2}$

$=\sqrt{\dfrac{53}{2}}$

$<\sqrt{27}$

The origin lies inside the circle.

6. A

I is true:

Note that the centre of the circle is $(1,0)$,

which showing that the centre lies on the x-axis.

II is not true.

III is not true:

Note that $(1)+0-3=-2\neq 0$.

The centre does not lie on the straight line $x+y-3=0$.

7. B

I is not true:

The radius of the circle $= \sqrt{\left(-\frac{\frac{8}{2}}{2}\right)^2 - \left(-\frac{9}{2}\right)} = \sqrt{\frac{17}{2}} \neq 5$.

II is true:

The coordinates of the mid-point of $PQ = \left(\frac{-4-1}{2}, \frac{0-5}{2}\right) = \left(-\frac{5}{2}, -\frac{5}{2}\right)$.

Note that $2\left(-\frac{5}{2}\right)^2 + 2\left(-\frac{5}{2}\right)^2 + 8\left(-\frac{5}{2}\right) - 9 = -4 < 0$.

Thus, the mid-point of PQ lies inside C.

III is not true:

The coordinates of G are $(0, -2)$.

Slope of $PG = \frac{-2-0}{0+4} = -\frac{1}{2}$

Slope of $QG = \frac{-2+5}{0+1} = 3$

$\angle PGQ = \tan^{-1} 3 + \tan^{-1}\left(\frac{1}{2}\right) \approx 98.13010235° > 90°$

So, $\angle PGQ$ is not an acute angle.

8. C

I is true:

Observe that the origin lies on C_1.

The coordinates of G_2 are $\left(-\frac{8}{5}, -\frac{6}{5}\right)$.

Note that $\left(-\frac{8}{5}\right)^2 + \left(-\frac{6}{5}\right)^2 - 14\left(-\frac{8}{5}\right) + 22\left(-\frac{6}{5}\right) = 0$.

So, G_2 lies on C_2.

Both OG_1 and G_1G_2 are radii of C_1.

Thus, ΔOG_1G_2 is an isosceles triangle.

II is not true:

$$G_1G_2 = \sqrt{\left(\frac{14}{2}\right)^2 + \left(-\frac{22}{2}\right)^2} = \sqrt{170}$$

The radius of C_2

$$= \sqrt{\left(-\frac{8}{5}\right)^2 + \left(-\frac{6}{5}\right)^2 + \left(\frac{825}{5}\right)}$$
$$= 13$$
$$< G_1G_2$$

So, G_1 lies outside C_2.

Thus, the line segment OG_1 does not lie inside C_2.

III is true:

Since G_2 lies on C_1 and the radius of $C_2 = 13 < G_1G_2$,

C_1 and C_2 intersect at two distinct points.

9. D

I is true:

The coordinates of the centre of C $= \left(\frac{8}{2}, \frac{2k}{2}\right) = (4, k)$.

II is true:

Note that $(-2)^2 + (1)^2 - 8(-2) - 2k(1) + 2k - 34 = -13 < 0$.

The point $(-2,1)$ lies inside C.

III is true:

Let A and r denote the area and the radius of C respectively.

$$
\begin{aligned}
r^2 &= 4^2 + k^2 - 2k + 34 \\
&= k^2 - 2k + 50 \\
&= (k-1)^2 + 49
\end{aligned}
$$

Note that $A = \pi r^2$. A is the least as r^2 attains its minimum.

So, the least possible area of C is 49π.

10. A

I is true:

The coordinates of $G_1 = \left(\frac{\frac{36}{4}}{2}, -\frac{\frac{48}{4}}{2}\right) = \left(\frac{9}{2}, -6\right)$.

The coordinates of $G_2 = \left(-\frac{18}{2}, \frac{24}{2}\right) = (-9, 12)$.

Slope of $OG_1 = \frac{-6-0}{\frac{9}{2}-0} = -\frac{4}{3}$

Slope of $OG_2 = \frac{12-0}{-9-0} = -\frac{4}{3}$

Since the slope of OG_1 equals to the slope of OG_2, G_1, G_2 and O are collinear.

II is not true:

The radius of $C_1 = \sqrt{\left(\frac{9}{2}\right)^2 + (-6)^2 - \left(\frac{125}{4}\right)} = 5$

The radius of $C_2 = \sqrt{(-9)^2 + 12^2 + 47} = 4\sqrt{17} \approx 16.4924225$

So, the radii of C_1 and C_2 are not equal.

III is not true:

Denote the radii of C_1 and C_2 by r_1 and r_2 respectively.

$$
\begin{aligned}
\text{The distance between } G_1 \text{ and } G_2 &= \sqrt{\left(-9-\frac{9}{2}\right)^2 + (12+6)^2} \\
&= 22.5 \\
&> 5 + 4\sqrt{17} = r_1 + r_2
\end{aligned}
$$

So, C_1 and C_2 do not intersect with each other.

11. B

Putting $y = 0$ into the equation of the circle, we have

$(x-5)^2 + (0-12)^2 = 169$

$x = 0$ or $x = 10$

Putting $x = 0$ into the equation of the circle, we have

$(0-5)^2 + (y-12)^2 = 169$

$y = 0$ or $y = 24$

So, the coordinates of A and B are $(10, 0)$ and $(0, 24)$ respectively.

The area of ΔOAB

$= \frac{(10)(24)}{2}$

$= 120$

12. B

Putting $y = 0$ into the equation of the circle, we have

$2x^2 - 16x + 32 = 0$

$x = 4$

Putting $x = 0$ into the equation of the circle, we have

$2y^2 - 16y + 32$

$y = 4$

So, the coordinates of A and B are $(4, 0)$ and $(0, 4)$ respectively.

Note that the circle touches the x-axis and y-axis at A and B respectively.

Thus, $AB = \sqrt{4^2 + 4^2} = 4\sqrt{2}$.

13. B

Putting $y = 0$ into the equation of the circle, we have

$(x+2)^2 + (0+3)^2 = 13$

$x = -4$ or $x = 0$

Putting $x = 0$ into the equation of the circle, we have

$(0+2)^2 + (y+3)^2 = 13$

$y = -6$ or $y = 0$

So, the coordinates of A and C are $(-4, 0)$ and $(0, -6)$ respectively.

The area of the rectangle $OABC$

$= (4)(6)$

$= 24$

14. A

Putting $y=0$ into the equation of the circle, we have

$3x^2-22x+39=0$

$x=3$ or $x=\frac{13}{3}$

Putting $x=0$ into the equation of the circle, we have

$3y^2+42y+39=0$

$y=-13$ or $y=-1$

So, the coordinates of P, R and S are $(3,0)$, $(0,-1)$ and $(0,-13)$ respectively.

The area of ΔPRS

$=\frac{(-1+13)(3)}{2}$

$=18$

15. D

The coordinates of the centre of the circle $=\left(\frac{\frac{12}{3}}{2},-\frac{\frac{30}{3}}{2}\right)=(2,-5)$.

Note that the straight line $kx+3y-10=0$ passes through the centre.

So, we have

$k(2)+3(-5)-10=0$

$k=\frac{25}{2}$

16. B

The coordinates of the centre of the circle $=\left(\frac{6}{2},\frac{6}{2}\right)=(3,3)$.

Note that $(3,3)$ is the mid-point of AB.

The coordinates of B

$=(3\times 2-7,3\times 2-9)$

$=(-1,-3)$

17. B

The coordinates of the centre of the circle $=\left(-\frac{2k}{2},\frac{10}{2}\right)=(-k,5)$.

So, we have

$\frac{-7-5}{4+k}=-12$

$k=-3$

18. B

Denote the centres of the circles $(x-2)^2+(y+2)^2=100$ and $(x+10)^2+(y-7)^2=625$ by G_1 and G_2 respectively.

Note that $G_1P=10$ and $G_2P=25$.

So, $G_2G_1:G_1P=3:2$.

The coordinates of P

$$=\left(\frac{(5)(2)-(2)(-10)}{3},\frac{(5)(-2)-(2)(7)}{3}\right)$$

$$=(10,-8)$$

Part 2A – Condition of equation of circle

1.	A	2.	B	3.	C	4.	D

1. A

The radius of the C $= \sqrt{(7-2)^2 + (11-5)^2} = \sqrt{61}$

The equation of C is

$(x-7)^2 + (y-11)^2 = 61$

$x^2 + y^2 - 14x - 22y + 109 = 0$

2. B

Let the coordinates of the centre of the circle be $(h, 0)$.

$(h-10)^2 + (0-8)^2 = (h+2)^2 + (0-4)^2$

$h = 6$

The equation of the circle is

$(x-6)^2 + y^2 = (6+2)^2 + (0-4)^2$

$(x-6)^2 + y^2 = 80$

$x^2 + y^2 - 12x - 44 = 0$

3. C

Let the coordinates of the centre of the circle be $(0, k)$.

$(0+8)^2 + (k-2)^2 = (0-20)^2 + (k+12)^2$

$k = -17$

The equation of the circle is

$x^2 + (y+17)^2 = (0+8)^2 + (-17-2)^2$

$x^2 + y^2 + 34y - 136 = 0$

4. D

Denote the points $(3,7)$ and $(-4,6)$ by A and B respectively.

The coordinates of the mid-point of $AB = \left(\frac{3-4}{2}, \frac{7+6}{2}\right) = \left(-\frac{1}{2}, \frac{13}{2}\right)$

Slope of $AB = \frac{6-7}{-4-3} = \frac{1}{7}$

Note that the line joining from the centre to the mid-point of AB is perpendicular to AB.

So, we have $\frac{2-\frac{13}{2}}{k+\frac{1}{2}} = -7$.

$\therefore\ k = \frac{1}{7}$

The equation of the circle is

$$\left(x-\frac{1}{7}\right)^2 + (y-2)^2 = \left(3-\frac{1}{7}\right)^2 + (7-2)^2$$

$$x^2 + y^2 - \frac{2}{7}x - 4y - \frac{204}{7} = 0$$

$$7x^2 + 7y^2 - 2x - 28y - 204 = 0$$