LOCUS

Form 5
Vol 5

Part 3C - Equation of Locus (先知先覺)

1. The equations of the locus

$$\left|\frac{3x+4y-1}{\sqrt{3^2+4^2}}\right|=3$$

$$3x+4y-1=15 \quad \text{and} \quad 3x+4y-1=-15$$
$$3x+4y-16=0 \qquad\qquad 3x+4y+14=0$$

2. Equation of AB

$$\frac{y+7}{x-2}=\frac{5+7}{-3-2}$$
$$-5(y+7)=12(x-2)$$
$$-5y-35=12x-24$$
$$12x+5y+11=0$$

The equations of locus

$$\left|\frac{12x+5y+11}{\sqrt{12^2+5^2}}\right|=\left|\frac{12(2)+5(10)+11}{\sqrt{12^2+5^2}}\right|$$

$$12x+5y+11=85 \quad \text{and} \quad 12x+5y+11=-85$$
$$12x+5y-74=0 \qquad\qquad 12x+5y+96=0$$

3. A

4. (a) (i) $\begin{cases} y = x^2 - 2x - 1 \\ y = x + 3 \end{cases}$

$x^2 - 2x - 1 = x + 3$

$x^2 - 3x - 4 = 0$

$\begin{cases} x = 4 \\ y = 7 \end{cases}$ or $\begin{cases} x = -1 \\ y = 2 \end{cases}$

$A(-1, 2)$, $B(4, 7)$ (or $A(4, 7)$, $B(-1, 2)$)

(ii) $y = x^2 - 2x - 1$

$= (x-1)^2 - 2$

$C(1, -2)$

(b) (i) It is the perpendicular bisector of AB.

(ii) The required equation is

$(x+1)^2 + (y-2)^2 = (x-4)^2 + (y-7)^2$

$2x + 1 - 4y + 4 = -8x + 16 - 14y + 49$

$x + y - 6 = 0$

(c) (i) Put $M(m, 2)$ into Γ

$\therefore m = 4$

(ii) The equation of the circle

$(x-4)^2 + (y-2)^2 = (1-4)^2 + (-2-2)^2$

$x^2 + y^2 - 8x - 4y - 5 = 0$

5. (a) Since C passes through A, we have

$$(-14+2)^2+(1+15)^2=k$$

$$k=400$$

Thus, the radius of C is 20.

(b) (i) The coordinates of G are $(-2,-15)$.

The slope of the line passes through A and G

$$=\frac{-15-1}{-2+14}$$

$$=-\frac{4}{3}$$

Let $N(-2,n)$ be the point lying on Γ such that $AN=12$.

$$\angle AGN=90°+\tan^{-1}\left(-\frac{4}{3}\right)$$

$$\approx 36.86989765°$$

$$\therefore\ n=-15+\frac{12}{\sin\angle AGN}=5$$

Thus, we have $N(-2,5)$.

The equation of Γ is

$$y-5=-\frac{4}{3}(x+2)$$

$$y=-\frac{4}{3}x+\frac{7}{3}$$

$$4x+3y-7=0$$

(ii) Note that Φ is the perpendicular bisector of BH.
Denote the mid-point of BH by M.

$$\begin{cases} 4x+3y-7=0 \\ 3x-4y+1=0 \end{cases}$$

By solving, we have $x=1,\ y=1$.
So, the coordinates of M are $(1,1)$.

The area of $\Delta HST = \frac{1}{2}(ST)(HM)$

The area of $\Delta KST = \frac{1}{2}(ST)(KM)$

Thus, the required ratio is $HM : KM$.
Note that HK is a chord of C.
Denote the mid-point of HK by N.

$$\begin{aligned} NH &= NK \\ &= \sqrt{20^2-12^2} = 16 \end{aligned}$$

$$GM = \sqrt{(1+2)^2+(1+15)^2} = \sqrt{265}$$

$$MN = \sqrt{GM^2-12^2} = 11$$

$$\begin{aligned} HM &= NH - MN \\ &= 16-11 \\ &= 5 \end{aligned}$$

$$\begin{aligned} KM &= NK + MN \\ &= 16+11 \\ &= 27 \end{aligned}$$

Therefore, the required ratio is $5:27$.

6. (a) Note that the slope of L_1 is $\frac{5}{4}$.

Denote the point where L_1 intersects the x-axis by $D(d,0)$.

$$\frac{30}{d+16}=\frac{5}{4}$$

$$d=8$$

Thus, the x-intercept of L_1 is 8.

$$AB=\sqrt{(20+16)^2+(15+30)^2}=9\sqrt{41}$$

The perpendicular distance from C to L_1

$$=\frac{1530\times 2}{9\sqrt{41}}$$

$$=\frac{340}{\sqrt{41}}$$

$$\angle ADC=\tan^{-1}\left(\frac{5}{4}\right)$$

$$\approx 51.34019175°$$

$$CD=\frac{\frac{340}{\sqrt{41}}}{\sin\angle ADC}=68$$

Thus, we have $C(-60,0)$.

The equation of Γ is

$$y=\frac{5}{4}(x+60)$$

$$5x-4y+300=0$$

(b) (i) Note that $\Delta CMN \sim \Delta BAN$.

Then we have $CN:BN=2:3$.

The coordinates of N

$$=\left(\frac{(2)(20)+(3)(-60)}{5},\frac{(2)(15)}{5}\right)$$

$$=(-28,6)$$

The slope of ℓ

$$=\frac{6+30}{-28+16}$$

$$=-3$$

The equation of ℓ is

$$y+30=-3(x+16)$$

$$3x+y+78=0$$

(ii) Note that the area of ΔABC is 1530.

Since $CN : BN = 2:3$, we have

Area of $\Delta CNA = 1530 \times \frac{2}{3+2} = 612$

Since $MN : AN = 2:3$, we have

Area of $\Delta CMN = 612 \times \frac{2}{3} = 408$