LOCUS

Form 5
Vol 5

Part 3B - Equation of Locus (先知先覺)

1. Distance

$$=\left|\frac{3(2)-4(3)+11}{\sqrt{3^2+(-4)^2}}\right|=1\text{ unit}$$

2. Equation of BC

$$\frac{y-1}{x-1}=\frac{2-1}{0-1}$$
$$y-1=-(x-1)$$
$$x+y-2=0$$

Distance of $BC=\sqrt{(1-0)^2+(1-2)^2}=\sqrt{2}$

$$\left|\frac{2t-t-2}{\sqrt{1^2+1^2}}\right|=\frac{6\times 2}{\sqrt{2}}$$

$$\therefore \begin{aligned} t-2&=12 \\ t&=14 \end{aligned} \quad\text{or}\quad \begin{aligned} t-2&=-12 \\ t&=-10 \end{aligned}$$

3. The equations of the locus

$$\begin{aligned} y-3&=x+2 \\ y&=x+5 \end{aligned} \quad\text{and}\quad \begin{aligned} y-3&=-(x+2) \\ y&=-x+1 \end{aligned}$$

4. The equations of the locus

$$\left|\frac{7x-y+12}{\sqrt{7^2+1^2}}\right|=\left|\frac{x+7y+1}{\sqrt{1^2+7^2}}\right|$$

$$\begin{aligned} 7x-y+12&=x+7y+1 \\ 6x-8y+11&=0 \end{aligned} \quad\text{and}\quad \begin{aligned} 7x-y+12&=-(x+7y+1) \\ 8x+6y+13&=0 \end{aligned}$$

5. The equations of the locus

$$\left|\frac{3x+4y-5}{\sqrt{3^2+4^2}}\right| = \left|\frac{12x+5y-7}{\sqrt{12^2+5^2}}\right|$$

$13(3x+4y-5)=5(12x+5y-7)$ $\quad$ $13(3x+4y-5)=-5(12x+5y-7)$

$39x+52y-65=60x+25y-35$ and $39x+52y-65=-60x-25y+35$

$7x-9y+10=0$ $\quad$ $99x+77y-100=0$

6. (a) (i) Γ is the angle bisector of $\angle OST$.

(ii) $OS=\sqrt{12^2+6^2}=6\sqrt{5}$

$TS=\sqrt{(-12+15)^2+6^2}=3\sqrt{5}$

Denote the point where Γ intersects the x-axis by N.

The area of $\Delta OSN=\frac{1}{2}(OS)(SN)\sin\angle OSN$

The area of $\Delta TSN=\frac{1}{2}(TS)(SN)\sin\angle TSN$

The ratio of the area of ΔOSN to the area of ΔTSN

$=OS:TS$

$=2:1$

So, we have $ON:TN=2:1$.

The coordinates of N

$=\left(\frac{2(-15)}{3},0\right)$

$=(-10,0)$

The equation of Γ is

$\frac{y}{x+10}=\frac{6}{-10+12}$

$3x-y+30=0$

Alternatively,

The slope of $L_1 = \dfrac{-6}{-12+15} = -2$

The slope of $L_2 = \dfrac{6}{12} = \dfrac{1}{2}$

$\therefore$ Slope of L_1 $\times$ Slope of $L_2 = -2\times\dfrac{1}{2} = -1$

$\therefore$ $L_1 \perp L_2$

Thus, we have $\angle OST = 90°$.

The slope of Γ

$= \tan\left(\dfrac{90°}{2} + \tan^{-1}\dfrac{1}{2}\right)$

$= 3$

The equation of Γ is

$y+6 = 3(x+12)$

$3x - y + 30 = 0$

(b) Denote the centre of the inscribed circle of ΔABS by $I(h,k)$.

Note that the angle bisector of $\angle ABS$ passes through I.

$\angle ABS = \tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{2}\right) = 2\tan^{-1}\left(\dfrac{1}{2}\right)$

$\angle IBS = \tan^{-1}\left(\dfrac{1}{2}\right)$

$=$ the inclination of L_2

So, the angle bisector of $\angle ABS$ is parallel to the x-axis.

The equation of L_2 is $y = \dfrac{1}{2}x$.

Putting $y = \dfrac{1}{2}x$ in $x + 2y = 24$, we have

$x + 2\left(\dfrac{1}{2}x\right) = 24$

$x = 12$

So, the coordinates of B are $(12,6)$.

Thus, we have $k = 6$.

Since I lies on Γ, we have

$3h - 6 + 30 = 0$

$h = -8$

Therefore, the coordinates of the centre of the inscribed circle of ΔABS are $(-8,6)$.

7. (a) Note that Γ is the perpendicular bisector of AB.

The slope of Γ

$= (-1) \times \left(\frac{4+18}{1-5} \right)$

$= \frac{11}{2}$

The coordinates of the mid-point of AB

$= \left(\frac{-18+4}{2}, \frac{5+1}{2} \right)$

$= (-7, 3)$

The equation of Γ is

$y - 3 = \frac{11}{2}(x+7)$

$11x - 2y + 83 = 0$

(b) (i) Note that Ψ is the angle bisector of $\angle BHD$.

Slope of $BH = \frac{-14-1}{24-4} = -\frac{3}{4}$

$\angle BHD = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{4}{3}\right) = 90°$

Slope of $\Psi = \tan\left[\tan^{-1}\left(\frac{4}{3}\right) - \frac{90°}{2}\right] = \frac{1}{7}$

The equation of Ψ is

$y + 14 = \frac{1}{7}(x - 24)$

$x - 7y - 122 = 0$

(ii) Note that Γ and Ψ pass through the centre of C.

The centre of C is the intersecting point of Γ and Ψ.

$$\begin{cases} 11x - 2y + 83 = 0 \\ x - 7y - 122 = 0 \end{cases}$$

By solving, we have $x = -11$, $y = -19$.

Thus, the coordinates of the centre of C are $(-11, -19)$.