# LOCUS

Form 5
Vol 5

**Part 3A - Equation of Locus (先知先覺)**

1. (a) $x^2+(y-k)^2=2^2+(5-k)^2$

$$x^2+y^2-2ky=4+25-10k$$
$$x^2+y^2-2ky+10k-29=0$$

(b) (i) $(x-2)^2+(y-5)^2=x^2+(y-k)^2$

$$-4x+4-10y+25=-2ky+k^2$$
$$4x+(10-2k)y+k^2-29=0$$

(b) (ii) Slope of $\Gamma=\dfrac{4}{2k-10}$

$$\frac{2-k}{1-0}\times\frac{4}{2k-10}=-1$$
$$8-4k=10-2k$$
$$k=-1$$

2. (a) Slope of $L_2=\dfrac{5}{2}$, let $L_2: y=\dfrac{5}{2}x+c$

put $(7,7)$ into $L_2$, $c=-\dfrac{21}{2}$

so, $L_2: y=\dfrac{5}{2}x-\dfrac{21}{2}$

(b) (i) It is the perpendicular bisector of $AB$.

(ii) put $x=0$ into $L_1$, $y=4$

therefore, locus of $P: y=-\dfrac{2}{5}x-\dfrac{13}{4}$



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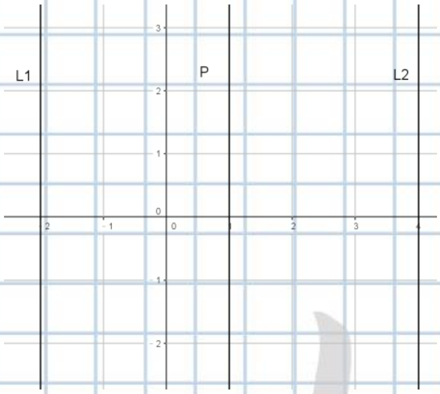
3. (a) It is a vertical line equidistant from them.




(b) $x=1$

4. $3x-y+8=0$

5. Slope $=\dfrac{1-0}{0+3}=\dfrac{1}{3}$

The equation of the locus of P :

$$y-0=\frac{1}{3}\left(x-\frac{-3+5}{2}\right)$$

$$y=\frac{1}{3}x-\frac{1}{3}$$



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6. (a) Note that $\Phi$ is a circle with centre $G$ and radius $GA$.

$GA=\sqrt{(7+23)^2+(-2-14)^2}=34$

The equation of $\Phi$ is

$(x+23)^2+(y-14)^2=1156$

$x^2+y^2+46x-28y-431=0$

(b) (i) Since $B$ lies on $\Phi$, we have

$(-39+23)^2+(k-14)^2=1156$

$k-14=-30$ or $k-14=30$

$k=-16$ or $k=44$ (rej.)

Note that $\Gamma$ is the perpendicular bisector of $AB$ and passes through $G$.

Slope of $\Gamma=(-1)\times\left(\dfrac{-39-7}{-16+2}\right)=-\dfrac{23}{7}$

The equation of $\Gamma$ is

$y-14=-\dfrac{23}{7}(x+23)$

$23x+7y+431=0$

(ii) The distance between $A$ and $B$

$=\sqrt{(-39-7)^2+(-16+2)^2}=34\sqrt{2}$

The distance between $H$ and $K$

$=2\times 34=68$

The area of the quadrilateral $AHBK$

$=\dfrac{68\times 34\sqrt{2}}{2}$

$=1156\sqrt{2}$



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7. (a) The slope of $ST$

$= \frac{17-2}{12+8}$

$= \frac{3}{4}$

The mid-point of $TR$

$= \left(\frac{12+6k^2}{2}, \frac{17-3-2k}{2}\right)$

$= (6+3k^2, 7-k)$

Note that $\Gamma$ is parallel to $L$ and $\ell$.

So, the equation of $\Gamma$ is

$y-(7-k) = \frac{3}{4}(x-6-3k^2)$

$3x-4y-9k^2-4k+10=0$

(b) (i) Note that either $SGR$ or $TGR$ is a straight line.

Case 1: $TGR$ is a straight line.

Then $TR$ is the diameter and thus $\angle RST = 90^\circ$.

slope of $RS = \frac{2+3+2k}{-8-6k^2} = -1 \div (\text{slope of } ST)$

$\frac{2k+5}{6k^2+8} = \frac{4}{3}$

$24k^2-6k+17=0$

$\Delta = 36-4(24)(17) = -1596 < 0$

$\therefore$ no solution of $k$

Case 2: $SGR$ is a straight line.

i.e. $SR$ is a diameter of C.

Thus, we have $ST \perp TR$.

$\frac{17+3+2k}{12-6k^2} = -\frac{4}{3}$

$4k^2-k-18=0$

$(4k-9)(k+2)=0$

$k = \frac{9}{4}$ (rej.) or $k=-2$



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(ii) Let $h$ be the perpendicular distance from $P$ to $L$.

The area of $\Delta GTK$

$= \frac{h}{2}(GT)$

The area of $\Delta QTH$

$=$ The area of $\Delta QGH$ $+$ The area of $\Delta GTH$

$= \frac{h}{2}(GH) + \frac{h}{2}(GH)$

$= h(GH)$

So, the required ratio is $2GH : GT$.

Note that $GH$ and $GT$ are radii of C.

Therefore, the required ratio is $2:1$.



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