超研數學 ASPIRE MATHEMATICS

SOLUTION

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REGULAR QUIZ 02

Form 5

Circle

Part A – MC (@3 marks)

1	B	BC and AC are chords, not arc.
2	C	$OH^2+AH^2=OA^2$ $(r-8)^2+\left(\frac{24}{2}\right)^2=r^2$ $r=13$ cm
3	C	Join BC $\angle CBD=180^\circ\times\frac{1}{4(5+3+1)}=5^\circ$ $\angle BCA=180^\circ\times\frac{5}{4(5+3+1)}=25^\circ$ $\angle BPC=180^\circ-5^\circ-25^\circ=150^\circ$ ($\angle$ sum of Δ) $\angle APD=\angle BPC=150^\circ$ (vert. opp. $\angle$s)
4	D	$\angle BAC=\frac{1}{2}\angle BOC=19^\circ$ ($\angle$ at centre twice $\angle$ at circumference) $\angle ACO=\angle BAC+\angle ADC=43^\circ$ (ext. $\angle$ of Δ) $\angle CEO=180^\circ-\angle ACO-\angle BOC=99^\circ$ ($\angle$ sum of Δ) $\angle AEB=\angle CEO=99^\circ$ (vert. opp. $\angle$)
5	D	Join CE It is trivial that $\Delta DAB\sim\Delta DCE$ $\frac{BC+10}{12}=\frac{15}{10}$ $BC=8$
6	B	$\angle ABC=90^\circ$ ($\angle$ in semi-circle) Join PQ $CP=CQ$ (tangent property) $\angle CPQ=45^\circ$ ($\angle$ sum of Δ) $\angle APQ=135^\circ$ (adj. $\angle$s on st. line) $\angle PMQ=\angle APQ=135^\circ$ ($\angle$ in alt. segment)

7	D	$\angle XZY = 180^\circ - x - y$ ($\angle$ sum of Δ) $\angle BXY = \angle BYX = 180^\circ - x - y$ ($\angle$ in alt. segment) $\angle ABC = 180^\circ - \angle BXY - \angle BYX$ ($\angle$ sum of Δ) $\angle ABC = 2x + 2y - 180^\circ$
8	B	$BD = BF$, $AE = AF$, $CD = CE = 5$ (tangent property) $AB + BC + AC = 32$ $AB + AD + CD + AE + CE = 32$ $AB + (BD + AE) + CD + CE = 32$ $AB + AB + 5 + 5 = 32$ $AB = 11$
9.	D	$\angle ABC = \angle ADC = 90^\circ$ ($\angle$ in semi-circle) $\angle DCE = 180^\circ - 90^\circ - 34^\circ = 56^\circ$ ($\angle$ sum of Δ) $\angle CBO = \angle ECD = 56^\circ$ (corr. $\angle$s, $OB // DC$) $OB = OC$ (radii) $\angle OCB = \angle OBC = 56^\circ$ (base $\angle$s, isos. Δ) $\angle BAC = 180^\circ - 90^\circ - \angle OCB = 34^\circ$ ($\angle$ sum of Δ)
10.	C	Join *OB* and *OC* $\angle OBP = 18^\circ$ (base $\angle$s, isos. Δ) $\angle BOQ = 18^\circ + 18^\circ = 36^\circ$ (ext. $\angle$ of Δ) $\angle COP = \angle OBP = 18^\circ$ ($\angle$ in alt. segment) $\angle COB = 180^\circ - 36^\circ - 18^\circ = 126^\circ$ (adj. $\angle$s on st. line) $\angle BAC = 180^\circ - \angle COB = 54^\circ$ (opp. $\angle$s, cyclic quad.)
11.	B	$\angle DAE = \angle ABD = 36^\circ$ ($\angle$ in alt. segment) $\angle BAD = 180^\circ - 36^\circ - 47^\circ = 97^\circ$ (adj. $\angle$s on st. line) $\angle BCD = 180^\circ - \angle BAD = 83^\circ$ (opp. $\angle$s, cyclic quad.)
12.	A	Denote *O* be centre and join *OP* and *OQ*. $OP \perp TP$ and $OQ \perp TQ$ (tangent $\perp$ radius) $\angle POQ = 360^\circ \times \frac{3}{3+5} = 135^\circ$ $\angle PTQ = 360^\circ - 90^\circ - 90^\circ - 135^\circ = 45^\circ$ ($\angle$ sum of polygon)

1.	B	2.	C	3.	C	4.	D	5.	D
6.	B	7.	D	8.	B	9.	D	10.	C
11.	B	12.	A						

Part B - Short Questions (14 marks)

1. (5 marks)

$\angle ACB = \angle ABC = 65°$ (base $\angle$s, isos. Δ)	1M
$\angle ACB + \angle ABC + \angle BAC = 180°$ ($\angle$ sum of Δ)	
$\angle BAC = 50°$	1A
$\angle BOC = 100°$ ($\angle$ at centre twice $\angle$ at circumference)	1A
$\angle OBC = \angle OCB$ (base $\angle$s, isos. Δ)	
$\angle OBC + \angle OCB + \angle BOC = 180°$ ($\angle$ sum of Δ)	
$\angle OBC = 40°$	1M
$\angle ABO = 65° - 40° = 25°$	1A
	(5)

2. (9 marks)

(a)	$\angle ABD = \angle ACB$ ($\angle$ in alt. segment)	1M
	$\angle AED = \angle ACB$ (corr. $\angle$s, $DE // BC$)	1M
	$\therefore \angle ABD = \angle AED$	
	$\therefore$ A, D, B and E are concyclic. (converse of $\angle$s in the same segment)	1
(b) (i)	$EA = EB$ (given) and $DA = DB$ (tangent properties)	
	$\angle EAB = \angle EBA$ and $\angle DAB = \angle DBA$ (base $\angle$s, isos. Δ)	
	$\angle ADE = \angle EBA$ ($\angle$s in the same segment)	1
	In ΔADE,	
	$\angle AED + \angle EAB + \angle DAB + \angle ADE = 180^\circ$ ($\angle$ sum of Δ)	
	$2\angle DAB + 2\angle ADE = 180^\circ$	
	$\angle DAB + \angle ADE = 90^\circ$	
	$\therefore \angle AFE = 90^\circ$ (ext. $\angle$ of Δ)	1
(ii)	$\angle ABC = \angle AFE = 90^\circ$ (corr. $\angle$s, $DE // BC$)	
	$\therefore AC$ is a diameter. (converse of $\angle$ in semi-circle)	1
	$\because EA = EB$ and $\angle AFE = 90^\circ$	
	$\therefore AF = FB$ (prop. of isos. Δ)	
	$\therefore AE = EC$ (intercept theorem)	
	Thus, E is the centre of circle ABC.	1

(iii) Note that $\Delta ADF \sim \Delta EAF$ (AAA)

$\frac{AF}{FD} = \frac{EF}{AF}$ (corr. sides, ~Δs)

$\frac{AF}{64} = \frac{36}{AF}$

$AF = 48$ 1A

Radius

$= \sqrt{EF^2 + AF^2}$

$= \sqrt{36^2 + 48^2}$

$= 60$ 1A

(9)