TRIGONOMETRY 3D

Form 6
Vol 3

Part 5B – Non Square Base

1. (a) Let D be a point lying on AC such that $VD \perp AC$.

Note that D is the mid-point of AC and $BD \perp AC$.

$VD^2 = VA^2 - AD^2$

$VD^2 = 20^2 - \left(\frac{24}{2}\right)^2$

$VD = 16 \text{ cm}$

Note that the required angle is $\angle VDB$.

$\cos\angle VDB = \frac{\text{the area of } \Delta ABC}{\text{the area of } \Delta VAC}$

$\cos\angle VDB = \frac{96}{\frac{1}{2}(24)(16)}$

$\angle VDB = 60°$

Therefore, the required angle is $60°$.

(b) Note that the area of $\Delta ABC = \frac{1}{2}(AC)(BD)$.

$BD = \frac{96}{\frac{1}{2}(24)}$

$BD = 8 \text{ cm}$

Note that the required angle is $\angle ABC$.

$\sin\angle ABD = \frac{AD}{BD}$

$\sin\angle ABD = \frac{\frac{24}{2}}{8}$

$\angle ABD \approx 56.30993247°$

$\angle ABC = 2\angle ABD$

$\angle ABC \approx 112.6198649°$

Therefore, the required angle is $113°$.

2. $AB=\sqrt{6^2+8^2}=10\text{ cm}$

$CD^2=8^2+12^2-2(8)(12)\cos 130^\circ$
$CD=18.2048\text{ cm}$

$AD=\sqrt{6^2+18.2048^2}=19.1681\text{ cm}$

By Heron's formula, area of $\Delta ABD=51.4590$

By **Area Projection Theorem**, required angle $=\cos^{-1}\dfrac{\text{area of }\Delta BCD}{\text{area of }\Delta ABD}=\cos^{-1}\dfrac{\frac{1}{2}(8)(12)\sin 130^\circ}{51.4590}=44.4^\circ$

3. (a) Note that $\angle BAD=120^\circ$ and $AD // BC$.

$\angle ABC=180^\circ-120^\circ=60^\circ$

Let E be a point lying on BC such that $VE\perp BC$ and $AE\perp BC$.

Note that $\angle VEA=45^\circ$.

$AE=AB\sin\angle ABC$

$AE=8\sin 60^\circ$

$AE=4\sqrt{3}\text{ cm}$

$VA=AE\tan 45^\circ$

$VA=4\sqrt{3}\text{ cm}$

(b) $AD^2=VD^2-VA^2$

$AD^2=(4\sqrt{6})^2-(4\sqrt{3})^2$

$AD=4\sqrt{3}\text{ cm}$

Let E be a point lying on CD produced such that $AE\perp CE$.

Note that $\angle ADE=60^\circ$.

$AE=AD\sin\angle ADE$

$AE=4\sqrt{3}\sin 60^\circ$

$AE=6$

Note that the required angle is $\angle VEA$.

$\tan\angle VEA=\dfrac{VA}{AE}$

$\tan\angle VEA=\dfrac{4\sqrt{3}}{6}$

$\angle VEA\approx 49.10660535^\circ$

Therefore, the required angle is 49.1°.

4. (a) $\because VA=VC=VB$

$\therefore \Delta VLA\cong\Delta VLC\cong\Delta VLB$

$LA=LC=LB$ (corr.sides, $\cong\Delta$s)

$\therefore$ L is the circumcentre of ΔABC

(b) Let X be a point on BC such that $AX \perp BC$

$\because$ L is the circumcentre of ΔABC

LX is a perpendicular bisector of BC

$\angle CAX = \sin^{-1}\dfrac{4}{12} = 19.4712°$

$\because LA = LC = LB$

$\therefore \angle LCA = 19.4712°$

$\angle CLX = 38.9424°$

Distance between L and $ABC = LX = \dfrac{CX}{\tan 38.9424°} = 4.9498 \approx 4.95\text{ cm}$

(c) Required angle $= \tan^{-1}\dfrac{12}{4.9498} = 67.6°$

(d) Let Y be a point on AB such that $VY \perp AB$

$LY = 6\tan 19.4712° = 2.12\text{ cm}$

Required angle $= \tan^{-1}\dfrac{12}{2.12} = 80.0°$

5. (a) Let O be the centre of the pentagon.

$\angle AOB = \dfrac{360}{5} = 72°$

$OA = \dfrac{0.5}{\sin 36°} = 0.8507\text{ cm}$

Required distance $= 0.8507 + \sqrt{0.8507^2 - 0.5^2} = 1.5390 \approx 1.54\text{ cm}$

(b) Let F be a point lying on BC produced such that $DF \perp BF$.

Note that $\angle DCF = 72°$.

$DF = DC\sin\angle DCF$

$DF = (1)\sin 72°$

$DF \approx 0.951056516\text{ cm}$

Note that the required angle is $\angle DCF$.

$\tan\angle DCF = \dfrac{VD}{DF}$

$\tan\angle DCF \approx \dfrac{2}{0.951056516}$

$\angle DCF \approx 64.56759182°$

Therefore, the required angle is $64.6°$.

(c) Extend AE and BC such that they meet at Y.

$\angle CDE = \dfrac{180-72°}{2}\times 2 = 108°$

$CE = 2(1)\sin 54° = 1.6180\text{ cm}$

Note that $\Delta YAB \sim \Delta YEC$.

$$\frac{YA}{YA+1}=\frac{1}{1.6180}\Rightarrow YA=1.6180\text{cm}$$

$\therefore \Delta YAB \cong \Delta DAB$

Let X be a point on AB such that $DX \perp AB$.

$\therefore YX = DX$

$$VY=\sqrt{(2\times 1.5390)^2+2^2}=3.6704\text{ cm}$$

Let Z be a point on VY such that $EZ \perp VY$ and $CZ \perp VY$.

In ΔVEY,

By Heron's formula, area of $\Delta EVY = 2.899$.

$$\frac{1}{2}EZ(3.6704)=2.8990$$

$$EZ=1.5796\text{cm}$$

By symmetry, $CZ = 1.5796$ cm

Required angle $=\cos^{-1}\dfrac{1.5796^2+1.5796^2-1.618^2}{2(1.5796)^2}=61.6^\circ$

睇完答案個樣:

6. (a) $\angle ADC = 180^\circ - 58^\circ = 122^\circ$

$$\frac{\sin\angle ACD}{AD}=\frac{\sin\angle ADC}{AC}$$

$$\frac{\sin\angle ACD}{37}=\frac{\sin 122^\circ}{56}$$

$\angle ACD \approx 34.07775728^\circ$ or $\angle ACD \approx 145.9222427$ (rej.)

$\angle CAD = 180^\circ - 122^\circ - \angle ACD \approx 23.92224272^\circ$

$CD^2 = CA^2 + AD^2 - 2(CA)(AD)\cos\angle CAD$

$CD^2 \approx 56^2 + 37^2 - 2(56)(37)\cos 23.92224272^\circ$

$CD \approx 26.77655058$ cm

≈ 26.8 cm

(b) (i) Let C' and D' be the points lying on the horizontal ground vertically below the vertices C and D respectively. Note that $CC' = 50$ cm.

$DD' = 37\sin 55^\circ \approx 30.30862564$ cm

$$\frac{DK}{CD+DK}=\frac{DD'}{CC'}$$

$$\frac{DK}{26.77655058+DK}=\frac{30.30862564}{50}$$

$DK \approx 41.21400734$ cm

≈ 41.2 cm

(ii) $\angle ADK = 180^\circ - 122^\circ = 58^\circ$

The area of ΔADK

$=\dfrac{1}{2}(AD)(DK)\sin\angle ADK$

$= \frac{1}{2}(37)(41.21400734)\sin 58°$

$\approx 646.6020185 \text{ cm}^2$

$\approx 647 \text{ cm}^2$

(iii) $AK^2 = AD^2 + DK^2 - 2(AD)(DK)\cos\angle ADK$

$AK^2 \approx 37^2 + (41.21400734)^2 - 2(37)(41.21400734)\cos 58°$

$AK \approx 38.09760181 \text{ cm}$

Consider the triangle ADK. Let F be the foot of the perpendicular from D to AK.

Area of $\Delta ADK = \frac{1}{2}(AK)(DF)$

$\frac{1}{2}(38.09760181)(DF) \approx 646.6020185$

$DF \approx 33.9444998 \text{ cm}$

Let θ be the inclination of the paper card $ABCD$ to the horizontal ground.

$\sin\theta = \frac{DD'}{DF}$

$\sin\theta \approx \frac{30.30862564}{33.9444998}$

$\theta \approx 63.23838474°$

Therefore, the inclination of the paper card $ABCD$ to the horizontal ground is $63.2°$.

Part 6 – Cut solid

1. (a) $EB=\dfrac{\sqrt{6^2+6^2}}{2}=\dfrac{\sqrt{72}}{2}=3\sqrt{2}\approx 4.24\text{ cm}$

$EC=\sqrt{(3\sqrt{2})^2+6^2}=3\sqrt{6}\approx 7.35\text{ cm}$

(b) 90°

2. B

3. C

4. (a) Let X be a point on CD such that $BX\perp CD$, and Y be a point on MN such that $BY\perp MN$.

In ΔBCX,

$BX=\sqrt{5^2-1^2}=\sqrt{24}\text{ cm}$

Reminder:
Prism -> *BCFG* is a rectangle

In $CDMN$,

$XY=CN=\dfrac{20}{5}\times 2=8\text{ cm}$

In ΔBCN,

$BN=\sqrt{5^2+8^2}=\sqrt{89}\text{ cm}$

In ΔBYN,

$BY=\sqrt{89-1^2}=\sqrt{88}\text{ cm}$

In ΔBXY,

$$\text{required angle}=\cos^{-1}\frac{8^2+(\sqrt{88})^2-(\sqrt{24})^2}{2(8)(\sqrt{88})}=31.5^\circ$$

(b) Let Z be a point on GH such that $FZ\perp GH$.

In ΔFGZ,

$FZ=\sqrt{5^2-1^2}=\sqrt{24}\text{ cm}$

$$\text{Required angle}=\tan^{-1}\frac{12}{\sqrt{24}}=67.8^\circ$$

(c) Extend DA and CB such that they meet at V, note that the required angle is $\angle DVC$.

$\dfrac{VA}{VD}=\dfrac{AB}{DC}$

$\dfrac{VA}{VA+5}=\dfrac{3}{5}$

$VA=7.5$

$$\sin\frac{\angle DVC}{2}=\frac{1.5}{7.5}$$

$\angle DVC = 23.1^{\circ}$

Thus, the required angle $= 23.1^{\circ}$.