TRIGONOMETRY 3D

Form 6
Vol 3

Part 4 - Pyramid

1. (a) $OA = \dfrac{\sqrt{12^2 + 12^2}}{2} = \dfrac{\sqrt{288}}{2}$

$$OV = \sqrt{20^2 - \left(\frac{\sqrt{288}}{2}\right)^2} = \sqrt{328} \approx 18.1$$

(b) Required angle $= \tan^{-1}\dfrac{\sqrt{328}}{6} \approx 71.7°$

(c) Let E be a point on AB such that $OE \perp AB$.
Let F be a point on VB such that $AF \perp VB$ and $CF \perp VB$

$$VE = \sqrt{328 + 6^2} = \sqrt{364}$$

$$\frac{1}{2}AF(20) = \frac{1}{2}(12)(\sqrt{364})$$

$$AF = \frac{3\sqrt{364}}{5}$$

$$AC = \sqrt{12^2 + 12^2} = \sqrt{288}$$

$$\text{Required angle} = 2\sin^{-1}\frac{\frac{1}{2}AC}{AF} = 2\sin^{-1}\frac{\frac{\sqrt{288}}{2}}{\frac{3}{5}\sqrt{364}} \approx 95.7°$$

4. (a) $VA=\dfrac{3}{\cos 70^\circ}=8.7714$

$OB=\dfrac{6\sin 60^\circ}{3}\times 2=2\sqrt{3}$

Required angle $=\cos^{-1}\dfrac{3.4641}{8.7714}=66.7^\circ$

(b) Let E be a point on AB such that $VE\perp AB$.

$VE=\sqrt{8.7714^2-3^2}=8.2424$

$VO=\sqrt{8.7714^2-3.4641^2}=8.0584$

Required angle $=\sin^{-1}\dfrac{8.0584}{8.2424}\approx 77.9^\circ$

(c) Let F be a point on VB such that $AF\perp VB$ and $CF\perp VB$.

$\dfrac{1}{2}(6)(8.2424)=\dfrac{1}{2}AF(8.7714)$

$AF=5.6381$

Required angle $=2\sin^{-1}\dfrac{\frac{1}{2}AC}{AF}=2\sin^{-1}\dfrac{3}{5.6381}\approx 64.3^\circ$

5. (a) $\angle AOB=\dfrac{360^\circ}{5}=72^\circ$

$OA=\dfrac{3}{\sin 36^\circ}=5.1039\text{ cm}$

$VO=\sqrt{10^2-5.1039^2}=8.5994\approx 8.60\text{ cm}$

(b) Let X be a point on CD such that $OX\perp CD$.

$OX=\dfrac{3}{\tan 36^\circ}=4.1291\text{ cm}$

Required angle $=\tan^{-1}\dfrac{8.5994}{4.1291}=64.4^\circ$

(c) Let Y be a point on VC such that $BY\perp CY$ and $DY\perp CY$.

$\dfrac{1}{2}(\sqrt{10^2-3^2})(6)=\dfrac{1}{2}BY(10)$

$BY=5.7236\text{cm}$

$\angle BCD=\dfrac{(180^\circ-72^\circ)}{2}\times 2=108^\circ$

$BD=2(6\sin 54^\circ)=9.7082\text{ cm}$

Required angle $=2\sin^{-1}\dfrac{\frac{1}{2}BD}{BY}=\sin^{-1}\dfrac{4.8541}{5.7236}=116^\circ$

7. (a) 90°

(b) Let X be a point on VB such that $AX \perp VB$ and $CX \perp VB$.

$$VA=\sqrt{9^2+9^2}=\sqrt{162}\text{ cm}$$

$$VB=\sqrt{162+9^2}=\sqrt{243}\text{ cm}$$

$$\frac{1}{2}(AX)(\sqrt{243})=\frac{1}{2}(9)(\sqrt{162})$$
$$AX=7.3485\text{cm}$$

$$AC=\sqrt{9^2+9^2}=\sqrt{162}\text{ cm}$$

$$\text{Required angle }=2\sin^{-1}\frac{\frac{1}{2}AC}{AX}=2\sin^{-1}\frac{\frac{1}{2}\sqrt{162}}{7.3485}=120^\circ$$

(c) 90°

Part 5A – Non Square Base

1. (a) Required angle $=\tan^{-1}\dfrac{5}{6}=39.8^\circ$

(b) Let X be a point on VB such that $CX \perp VB$, and Y be a point on AB such that $YX \perp VB$.

$$VC=\sqrt{5^2+8^2}=\sqrt{89}$$

$$VB=\sqrt{VC^2+6^2}=\sqrt{125}$$

$$CX=\frac{6\sqrt{89}}{\sqrt{125}}$$

$$BX=\sqrt{6^2-CX^2}=\frac{36}{\sqrt{125}}$$

$$VA=\sqrt{5^2+6^2}=\sqrt{61}$$

$$XY=\frac{(VA)(BX)}{AB}=\frac{9\sqrt{61}}{2\sqrt{125}}$$

$$BY^2=BX^2+XY^2=\frac{81}{4}$$

$$CY=\sqrt{BY^2+6^2}=7.5$$

$$\cos\angle CXY=\frac{CX^2+XY^2-CY^2}{2(CX)(XY)}$$
$$\angle CXY=130.6510409^\circ$$

Required angle $\angle CXY = 131^\circ$ (cor. to 3 sig. fig.)

(c) Required angle is $90°$

2. Let X be a point on AB such that $CX \perp AB$, Y be a point on DB such that $XY \perp AB$.

In ΔABC,

By Heron's formula, area of $\Delta ABC \approx 26.906\,\text{cm}^2$

$$\frac{1}{2}(AB)(CX) = 26.906$$

$$CX \approx 4.4843$$

$$BX^2 = BC^2 - CX^2$$

$$BX = 5.375$$

In ΔABD,

$$\cos\angle ABD = \frac{12^2 + 10^2 - 8^2}{2(12)(10)}$$

$$\angle ABD \approx 41.410^\circ$$

$$\tan\angle ABD = \frac{XY}{BX}$$

$$XY \approx 4.7403$$

$$BY^2 = BX^2 + XY^2$$

$$BY \approx 7.1667$$

In ΔBCD,

$$\cos\angle CBD = \frac{7^2 + 10^2 - 8^2}{2(7)(10)}$$

$$\angle CBD \approx 52.617^\circ$$

$$CY^2 = BY^2 + 7^2 - 2(BY)(7)\cos\angle CBD$$

$$CY \approx 6.2805$$

In ΔCXY,

$$\cos\angle CXY = \frac{CX^2 + XY^2 - CY^2}{2(CX)(XY)}$$

$$\angle CXY \approx 85.8^\circ$$

Thus, the required angle is 85.8°.

3. Let X be a point lying on AC such that $DX \perp AC$.

Let Y be a point lying on BC such that $YX \perp AC$.

Note that the required angle is $\angle DXY$.

$$\cos\angle ACD = \frac{AC^2 + CD^2 - AD^2}{2(AC)(CD)}$$

$$\cos\angle ACD \approx \frac{20^2 + 13^2 - 15^2}{2(20)(13)}$$

$$\angle ACD \approx 48.58268958°$$

$$DX = CD\sin\angle ACD$$

$$DX \approx 13\sin 48.58268958°$$

$$DX \approx 9.748846086\,\text{cm}$$

$CX = CD\cos\angle ACD$

$CX \approx 13\cos 48.58268958°$

$CX = 8.6 \text{ cm}$

$$\cos\angle ACB = \frac{AC^2 + CB^2 - AB^2}{2(AC)(CB)}$$

$$\cos\angle ACB = \frac{20^2 + 26^2 - 15^2}{2(20)(26)}$$

$\angle ACB \approx 35.08808946°$

$XY = CX \tan\angle ACB$

$XY \approx 8.6\tan 35.08808946°$

$XY \approx 6.041510821 \text{ cm}$

$$CY = \frac{CX}{\cos\angle ACB}$$

$$CY \approx \frac{8.6}{\cos 35.08808946°}$$

$CY \approx 10.50998825 \text{ cm}$

$$\cos\angle BCD = \frac{BC^2 + CD^2 - BD^2}{2(BC)(CD)}$$

$$\cos\angle BCD = \frac{26^2 + 13^2 - 18^2}{2(26)(13)}$$

$\angle BCD \approx 39.58230557°$

$DY^2 = DC^2 + CY^2 - 2(DC)(CY)\cos\angle BCD$

$DY^2 \approx 13^2 + 10.50998825^2 - 2(13)(10.50998825)\cos 39.58230557°$

$DY \approx 8.29794298 \text{ cm}$

$$\cos\angle DXY = \frac{DX^2 + XY^2 - DY^2}{2(DX)(XY)}$$

$$\cos\angle DXY \approx \frac{9.748846086^2 + 6.041510821^2 - 8.29794298^2}{2(9.748846086)(6.041510821)}$$

$\angle DXY \approx 57.84967406°$

Therefore, the required angle is $57.8°$

4. (a) $\angle ABD = 180° - 90° - 50 = 40°$

$\angle BCD = 180° - 60° - 40° = 80°$

$$\frac{BD}{\sin\angle BCD} = \frac{CD}{\sin\angle CBD}$$

$$\frac{BD}{\sin 80°} = \frac{50}{\sin 60°}$$

$BD \approx 56.85790213 \text{ cm}$

$\approx 56.9 \text{ cm}$

$AD = BD\cos\angle ADB$

$AD \approx 56.85790213\cos 50°$

$AD \approx 36.547555 \text{ cm}$

$\approx 36.5 \text{ cm}$

(b) (i) $\cos\angle ADC = \dfrac{AD^2 + DC^2 - AC^2}{2(AD)(DC)}$

$\cos\angle ADC \approx \dfrac{(36.547555)^2 + 50^2 - 29^2}{2(36.547555)(50)}$

$\angle ADC \approx 34.97475531°$

$\approx 35.0°$

(ii) In Figure 3(b), let X be the foot of the perpendicular from A to BD. AX produced meets BD at Y.

Note that the angle between the plane ABD and the plane BCD is $\angle AXY$ in Figure 3(b).

$AX = AD\sin\angle ADB$

$AX \approx 36.547555\sin 50°$

$\approx 27.99705142 \text{ cm}$

$DX = AD\cos\angle ADB$

$DX \approx 36.547555\cos 50°$

$\approx 23.49231552 \text{ cm}$

Note that $\angle BDC = 40°$.

$XY = DX\tan\angle BDC$

$XY \approx 23.49231552\tan 40°$

$\approx 19.71239329 \text{ cm}$

$DY = \dfrac{DX}{\cos\angle BDC}$

$DY \approx \dfrac{23.49231552}{\cos 40°}$

$\approx 30.66703992 \text{ cm}$

$AY^2 = AD^2 + DY^2 - 2(AD)(DY)\cos\angle ADC$

$AY^2 \approx 36.547555^2 + 30.66703992^2 - 2(36.547555)(30.66703992)\cos 34.97475531°$

$AY \approx 20.96198564 \text{ cm}$

$\cos\angle AXY = \dfrac{AX^2 + XY^2 - AY^2}{2(AX)(XY)}$

$\cos\angle AXY \approx \dfrac{27.99705142^2 + 19.71239329^2 - 20.96198564^2}{2(27.99705142)(19.71239329)}$

$\angle AXY \approx 48.38740011°$

$> 48°$

$\therefore$ The claim is disagreed.