

REGULAR QUIZ 01

Form 6
3D Trigonometry

Part A - MC (@3 marks)

1.	A	$AD = A'D = \sqrt{1.2^2 + 0.8^2} = \sqrt{2.08}$ $AA'^2 = AB^2 + A'B^2 - 2(AB)(A'B)\cos 50^\circ$ $AA' \approx 1.014283828$ $\cos \theta = \frac{AD^2 + A'D^2 - AA'^2}{2(AD)(A'D)}$ $\theta = 41.2^\circ$
2.	D	<p>Let X be a point on BE such that $CX \perp BE$</p> <p>Note that X lies on BM</p> <p>The required angle is $\angle CKX$</p> <p>\therefore None of the above</p>
3.	D	<p>Denote the midpoint of CD by M. Note that $NM \perp CD$.</p> $MN = \frac{1}{2} BE = 4.5 \text{ cm}$ $AM = \sqrt{4^2 + 12^2} = \sqrt{160} \text{ cm}$ $\tan \angle NAM = \frac{NM}{AM} = \frac{4.5}{\sqrt{160}}$ $\angle NAM \approx 19.58333291^\circ$ $\angle NAM \approx 19.6^\circ$

4.	D	<p>Let $2a$ be the side length of the tetrahedron.</p> $4 \times \frac{1}{2} (2a)(2a) \sin 60^\circ = 562$ $a^2 = \frac{140.5}{\sqrt{3}}$ <p>Height of the tetrahedron</p> $= \sqrt{(\sqrt{3}a)^2 - \left(\frac{\sqrt{3}}{3}a\right)^2} = \sqrt{\frac{8}{3}}a$ <p>Volume</p> $= \frac{1}{3} \left(\frac{562}{4}\right) \left(\sqrt{\frac{8}{3}}a\right) = 689 \text{ cm}^3$
5.	A	<p>Let D be a point on AC such that $BD \perp AC$</p> $\tan \theta = \frac{VB}{BD} = 3$ $BD = \frac{8}{3}$ $AD = \sqrt{15^2 - \left(\frac{8}{3}\right)^2} = \frac{\sqrt{1961}}{3}$ <p>Note that $\triangle ABC \sim \triangle ADB$</p> $\frac{AC}{AB} = \frac{AB}{AD}$ $AC = \frac{675}{\sqrt{1961}}$ <p>Area</p> $= \frac{1}{2} (AC)(BD)$ $= 20.3 \text{ cm}^2$

6. B Let X be a point on VB such that $CX \perp VB$
 and Y be a point on AB such that $XY \perp VB$
 By Heron's formula, area of $\Delta VBC = 28.618$

$$CX = \frac{2(28.618)}{10} = 5.7236$$

$$BX = \sqrt{6^2 - CX^2} = 1.8$$

$$\cos \angle VBA = \frac{10^2 + 8^2 - 10^2}{2(10)(8)}$$

$$\cos \angle VBA = 0.4$$

$$\angle VBA = 66.422^\circ$$

$$\tan \angle VBA = \frac{XY}{BX}$$

$$XY = 4.1243$$

$$\cos \angle VBA = \frac{BX}{BY}$$

$$BY = 4.5$$

$$CY^2 = 6^2 + BY^2 = 56.25$$

$$\cos \theta = \frac{CX^2 + XY^2 - CY^2}{2(CX)(XY)}$$

$$\theta = 97.889^\circ$$

$$\therefore \text{Required angle} = 97.9^\circ$$

1. A 2. D 3. D 4. D 5. A
 6. B

Part B - Long Questions (39 marks)

1. (10 marks)

(a) $MB = 48 \times \frac{3}{2+3} = 28.8$ 1M

$\cos \angle VBA = \frac{15}{48} = \frac{5}{16}$ 1M

$MA = \sqrt{AB^2 + MB^2 - 2(AB)(MB)\cos \angle VBA}$ 1M

$MA \approx 34.48825887$

$MA = 34.5$ 1A r.t. 34.5

(b) $\frac{MN}{BC} = \frac{2}{2+3}$

$MN = 12$ 1A

Height of trapezium $MNDA$

$= \sqrt{MA^2 - \left(\frac{DA - MN}{2}\right)^2}$ 1M (either)

≈ 33.29324256

$= 33.3$ 1A

Height of trapezium $MNCB$

$= \sqrt{MB^2 - \left(\frac{BC - MN}{2}\right)^2}$ 1M (either)

≈ 27.35763148

$= 27.4$ 1A

(c) Let θ be the required angle.

$\cos \theta = \frac{33.293^2 + 30^2 - 27.358^2}{2(33.293)(30)}$ 1M

$\theta \approx 50.89388506$

\therefore Required angle $= 50.9^\circ$ 1A r.t. 50.9°

2. (6 marks)

(a) Let Q be a point on VB such that $AQ \perp VB$.

It is trivial that $\Delta VAB \cong \Delta VCB$

$\therefore CQ \perp VB$

$$\text{Put } s = \frac{12+12+8}{2} = 16$$

By Heron's formula, area of ΔVAB

$$= \sqrt{16(16-12)(16-12)(16-8)} \quad 1M$$

$$= \sqrt{2048}$$

$$AQ = \frac{2(\sqrt{2048})}{12} = \frac{16\sqrt{2}}{3} \quad 1A$$

$$\therefore CQ = AQ = \frac{16\sqrt{2}}{3}$$

$$\cos \angle AQC = \frac{AQ^2 + CQ^2 - AC^2}{2(AQ)(CQ)} \quad 1M$$

$$\therefore \angle AQC \approx 64.05552023^\circ$$

Required angle = 64.1° 1A r.t. 64.1°

(b) $\angle ABC = 60^\circ$

$$\cos \angle AVC = \frac{12^2 + 12^2 - 8^2}{2(12)(12)}$$

$$\angle AVC \approx 38.94244127^\circ$$

$$\text{Note that } \sin \frac{\angle APC}{2} = \frac{\frac{1}{2}AC}{AP}$$

Since $AP \geq AQ$, we have $\angle APC \leq \angle AQC$ 1M

$\angle APC$ increases as P moves from B (60°) to Q (64.1°) 1A for correctly describe (f.t.)
and decreases as P moves from Q (64.1°) to V (38.9°).

3. (12 marks)

(a) $BH = DH = 18 \cos 70^\circ$ 1M
 BD

$$= \sqrt{BH^2 + DH^2}$$

$$= \sqrt{2(18 \cos 70^\circ)^2} \quad 1M$$

$$\approx 8.706411455 \text{ cm}$$

$$\approx 8.71 \text{ cm} \quad 1A$$

(b) Area of $\triangle ACD$

$$= \frac{1}{2} (18)^2 \sin 140^\circ \quad 1M$$

$$\approx 104.1315928 \text{ cm}^2$$

Volume of the tetrahedron $ABCD$

$$= \frac{1}{3} \times \text{area of } \triangle ACD \times BH$$

$$\approx \frac{1}{3} \times 104.13 \times 18 \cos 70^\circ \quad 1M$$

$$\approx 213.6906137 \text{ cm}^3$$

$$\approx 214 \text{ cm}^3 \quad 1A$$

(c) Let h cm be the vertical distance from A to the horizontal ground, and k cm be the perpendicular distance from A to DC .

$$\text{Put } s = \frac{BD + BC + CD}{2} \approx 22.35320573$$

By Heron's formula, area of $\triangle BCD$

$$= \sqrt{s(s - BD)(s - BC)(s - CD)}$$

$$= \sqrt{22.353(22.353 - 8.7064)(22.353 - 18)(22.353 - 18)} \quad 1M$$

$$\approx 76.03165111 \text{ cm}^2$$

$$\frac{1}{3} \times 76.032 \times h = 213.69 \quad 1M$$

$$h \approx 8.431644345$$

$$k = 18 \sin 40^\circ \quad 1M$$

The required angle θ

$$\sin \theta = \frac{h}{k} \quad 1M$$

$$\theta \approx 46.78082111^\circ$$

$$\theta \approx 46.8^\circ \quad 2A$$

The required angle = 46.8°

4(a). (11 marks)

$$(a)(i) \quad CD = \sqrt{10^2 - 8^2} = 6 \quad 1M$$

$$BC = \sqrt{8^2 - 6^2} = 2\sqrt{7}$$

By Heron's formula, $1M$

$$s = \frac{8+6+2\sqrt{7}}{2}$$

Area of $\triangle CAB$

$$= \sqrt{s(s-8)(s-6)(s-2\sqrt{7})}$$

$$\approx 15.87450787$$

$$= 15.9 \text{ cm}^2$$

$1A \text{ (or } 6\sqrt{7} \text{ cm}^2)$

(ii) Volume of pyramid

$$= \frac{1}{3}(15.87450787)(6) \quad 1M$$

$$\approx 31.74901573$$

$$= 31.7 \text{ cm}^3 \quad 1A \text{ r.t. } 31.7$$

$$(iii) \quad \text{Area of } \triangle ABD = \frac{8 \times 6}{2} = 24 \quad 1M$$

Shortest distance

$$= \frac{(31.74901573)(3)}{24} \quad 1M$$

$$\approx 3.968626967$$

$$= 3.97 \text{ cm} \quad 1A \text{ r.t. } 3.97$$

$$(iv) \quad BC^2 + AB^2 = (2\sqrt{7})^2 + 6^2 = 64 \quad 1M$$

$$AC^2 = 8^2 = 64$$

$$\therefore BC \perp AB \text{ and } DB \perp AB \quad 1M$$

$$\therefore \text{yes, agree.} \quad 1A \text{ f.t.}$$

4(b). bonus (3 marks)

$$DC = \sqrt{10^2 - 6^2} = 8$$

$$BC = \sqrt{8^2 - 8^2} = 0 \quad 1M$$

∴ It is not possible that $AC = 6 \text{ cm}$ 2A f.t.

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