CIRCLE

Form 5
Vol 4

Part 6 – Tangent properties

1.	A	2.	B	3.	A	4.	A	5.	C
6.	B	7.	C	8.	D	9.	D	10.	A
11.	D	12.	C	13.	B				

1. A

Join *AC*.

$DA = DC$ (tangent properties)

$\angle DAC = \angle DCA$ (base $\angle$s, isos. Δ)

$\angle DAC + \angle DCA + \angle ADC = 180^\circ$ ($\angle$ sum of Δ)

$2\angle DAC + 45^\circ = 180^\circ$

$\angle DAC = 67.5^\circ$

$y = \angle DAC = 67.5^\circ$ ($\angle$ in alt. segment)

2. B

$TA = TB$ (tangent properties)

$\angle TAB = \angle TBA$ (base $\angle$s, isos. Δ)

$\angle TAB + \angle TBA + \angle ATB = 180^\circ$ ($\angle$ sum of Δ)

$2\angle TAB + 80^\circ = 180^\circ$

$\angle TAB = 50^\circ$

$x = \angle TAB = 50^\circ$ ($\angle$ in alt. segment)

3. A

$\angle CBA : \angle CAB = 2:1$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle CBA = 2\angle CAB = 70^\circ$

$\angle ACB + \angle CBA + \angle CAB = 180^\circ$ ($\angle$ sum of Δ)

$\angle ACB + 70^\circ + 35^\circ = 180^\circ$

$\angle ACB = 75^\circ$

$\angle TAB = \angle TBA = \angle ACB = 75^\circ$ ($\angle$ in alt. segment)

$\angle ATB + \angle TAB + \angle TBA = 180^\circ$ ($\angle$ sum of Δ)

$\angle ATB + 75^\circ + 75^\circ = 180^\circ$

$\angle ATB = 30^\circ$

4. A

Join AB.

$\angle BAC = \angle CBT$ and $\angle ABC = \angle CAT$ ($\angle$ in alt. segment)

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$ ($\angle$ sum of Δ)

$\angle BAC + \angle ABC + 110^\circ = 180^\circ$

$\angle BAC + \angle ABC = 70^\circ$

$\angle ATB + \angle TAB + \angle TBA = 180^\circ$ ($\angle$ sum of Δ)

$\angle ATB + (\angle BAC + \angle CAT) + (\angle ABC + \angle CBT) = 180^\circ$

$\angle ATB + 2\angle BAC + 2\angle ABC = 180^\circ$

$\angle ATB + 2(70^\circ) = 180^\circ$

$\angle ATB = 40^\circ$

5. C

$\angle BAO = \angle CAO = 35^\circ$ (tangent properties)

$\angle ACF = \angle BAC = 70^\circ$ (alt. $\angle$s, $BA // DF$)

6. B

$\angle DCO = \angle ECO$ (tangent properties)

$\angle BAC + \angle ACD = 180^\circ$ (int. $\angle$s, $BA // DF$)

$2\angle CAO + 2\angle ECO = 180^\circ$

$\angle CAO + \angle ECO = 90^\circ$

$\angle AOC + \angle CAO + \angle ECO = 180^\circ$ ($\angle$ sum of Δ)

$\angle AOC + 90^\circ = 180^\circ$

$\angle AOC = 90^\circ$

7. C

$AP = AR = 8$ cm (tangent properties)

$BQ = BP$ (tangent properties)

$BQ = AB - AP = 20 - 8 = 12$ cm

8. D

Denote the intersection of AC and the circle by P, and the intersection of BC and the circle by Q.

$BQ = BR$, $AR = AP$ and $CP = CQ$ (tangent properties)

$AC = \sqrt{AB^2 + BC^2} = \sqrt{12^2 + 16^2} = 20$ cm

$AP + CP = AC$

$AR + CQ = AC$

$(12 - BR) + (16 - BQ) = 20$

$12 + 16 - 2BR = 20$

$2BR = 8$

$BR = 4$ cm

9. D

Join AB.

$TA = TB$ (tangent properties)

$\angle TBA = \angle TAB$ (base $\angle$s, isos. Δ)

$\angle TBA + \angle TAB + \angle ATB = 180^\circ$ ($\angle$ sum of Δ)

$2\angle TBA + 58^\circ = 180^\circ$

$\angle TBA = 61^\circ$

$\angle CBA = \angle TBA - \angle CBT = 61^\circ - 26^\circ = 35^\circ$

$\angle CAT = \angle CBA = 35^\circ$ ($\angle$ in alt. segment)

10. A

$\angle QOR = 2\angle QPR = 2\times 58^\circ = 116^\circ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle OBC + \angle BOC + \angle OCB = 180^\circ$ ($\angle$ sum of Δ)

$\angle OBC + 116^\circ + 34^\circ = 180^\circ$

$\angle OBC = 30^\circ$

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$ ($\angle$ sum of Δ)

$\angle BAC + 2\angle OBC + 2\angle OCB = 180^\circ$ (incentre)

$\angle BAC + 2(30^\circ) + 2(34^\circ) = 180^\circ$

$\angle BAC = 52^\circ$

11. D

I is true.

$TA = TB$ (tangent properties)

II is true.

$\because \Delta TAB$ is right and isos.

$\therefore \angle TAB = 45^\circ$

$\angle CDB = \angle TAB = 45^\circ$ ($\angle$ in alt. segment)

$\angle CBD = 45^\circ$ ($\angle$ sum of Δ)

$\because \angle CDB = \angle CBD = 45^\circ$

$\therefore \Delta BCD$ is isos. (sides opp. eq. $\angle$s)

III is true

$\angle BCA = 90^\circ$

12. C

Join AB and BC.

Let $\angle ABR = x$

$\angle ACB = \angle ABP = 50^\circ + x$ ($\angle$ in alt. segment)

$\angle CAB = \angle BRC + \angle ABR = 29^\circ + x$ (ext. $\angle$ of Δ)

Note that $\Delta APC \cong \Delta BPC$ (SAS)

$\therefore CA = CB$ (corr. sides, $\cong \Delta$s)

$\angle ABC = \angle CAB = 29^\circ + x$ (base $\angle$s, isos. Δ)

$\angle ABC + \angle CAB + \angle ACB = 180^\circ$ ($\angle$ sum of Δ)

$(29^\circ + x) + (29^\circ + x) + (50^\circ + x) = 180^\circ$

$x = 24^\circ$

$\angle QBC = \angle QCB = \angle BAC = 53^\circ$ ($\angle$ in alt. segment)

$\angle BQC = 74^\circ$ ($\angle$ sum of Δ)

13. B

$RC = RB$ (tangent properties)

$\angle BCR = \angle CBR = 71^\circ$ (base $\angle$s, isos. Δ)

$\angle SBC + \angle BSR = \angle BCR$ (ext. $\angle$ of Δ)

$\angle SBC + 38^\circ = 71^\circ$

$\angle SBC = 33^\circ$

$$\begin{cases} \angle CBT + \angle TBR = 71^\circ \\ \angle CBT + 33^\circ = \angle TBR \text{ } (\angle \text{ bisector}) \end{cases}$$

$\therefore \angle CBT = 19^\circ$ and $\angle TBR = 52^\circ$

$\angle BPA = \angle TBR = 52^\circ$ (corr. $\angle$s, $BT // PQ$)

$PA = PB$ (tangent properties)

$\angle PAB = \angle PBA$ (base $\angle$s, isos. Δ)

$\angle PAB + \angle PBA + \angle BPA = 180^\circ$ ($\angle$ sum of Δ)

$2\angle PAB + \angle PBA + 52^\circ = 180^\circ$

$\angle PAB = 64^\circ$

$\angle TBA = \angle PAB = 64^\circ$ (alt. $\angle$s, $BT // PQ$)

$\therefore \angle ABS = 12^\circ$

Part 7 – Tangent (SQ)

2. (a) $\angle APC = 72°$ ($\angle$s in alt. segment)

$\angle OPC = 90°$ (tangent $\perp$ radius)

$\angle OPA = 90° - 72° = 18°$

(b) $BT = BP$ (given)

$\angle BTP = \angle BPT$ (base $\angle$s, isos. Δ)

$2\angle BPT = 72°$ (ext. $\angle$ of Δ)

$\angle BPT = 36°$

$\angle PAB = \angle BPT = 36°$ ($\angle$s in alt. segment)

6. Let $BQ = 2k$ and $QC = 5k$.

$PB = BQ = 2k$ (tangent prop.)

$AP = 16 - 2k$

$RC = QC = 5k$ (tangent prop.)

$AR = 22 - 5k$

$16 - 2k = 22 - 5k$ (tangent prop.)

$k = 2$

$BC = 7(2) = 14$ cm

Part 8 – Proving problem - concyclic

5. (a) Let $\angle BAR = \angle ADB = y$

$\angle ACB = \angle ADB = y$ ($\angle$s in the same segment)

$\angle BAC = 90°$ ($\angle$ in semi-circle)

$\therefore \angle RAC = 90° - y$

In ΔARC,

$90° - y + y + \angle ARC = 180°$ ($\angle$ sum of Δ)

$\angle ARC = 90°$

$\therefore AR \perp BC$

(b) $\angle BDC = 90°$ ($\angle$ in semi-circle)

$\therefore \angle QDC = 90°$

$\angle QRC = 90°$ (by (a))

$\because \angle QDC + \angle QRC = 90° + 90° = 180°$

$\therefore C, D, Q$ and R are concyclic. (opp. $\angle$s supp.)

(c) $\angle BAR + \angle ABD = \angle BQR$ (ext. $\angle$ of Δ)

$25° + \angle ABD = 70°$

$\angle ABD = 45°$

7. (a) $OC \perp TC$ (tangent $\perp$ radius)

$\angle OCT = 90°$

$OA \perp TA$ (tangent $\perp$ radius)

$\angle OAT = 90°$

$\because \angle OCT + \angle OAT = 90° + 90° = 180°$

$\therefore O, A, T$ and C are concyclic. (opp. $\angle$s supp.)

(b) $\angle ATO = \angle CTO = 35°$ (tangent prop.)

$\therefore \angle ATC = 35° + 35° = 70°$

(c) Since O, A, T and C are concyclic,

$\angle AOC + \angle ATC = 180°$ (opp. $\angle$s cyclic quad.)

$\angle AOC + 70° = 180°$

$\angle AOC = 110°$

$\angle ABC = \frac{1}{2} \times 110° = 55°$ ($\angle$ at centre twice $\angle$ at circumference)

9. $\because \widehat{BC} = \widehat{QR}$ (given)

$\therefore \angle YAZ = \angle YPZ$ (equal arcs, equal angles)

$\therefore PYZA$ is a cyclic quadrilateral. (converse of $\angle$s in the same segment)

10. $\angle BCA = \angle AED = 90°$ ($\angle$ in semi-circle)

$\angle ACP = 90°$ (adj. $\angle$s on st. line)

$\angle PEA = 90°$ (adj. $\angle$s on st. line)

$\because \angle ACP + \angle PEA = 90° + 90° = 180°$

$\therefore$ $ACPE$ is a cyclic quadrilateral. (opp. $\angle$s supp.)

11. Join OA,

$\angle OAB = \angle ABC$ (base $\angle$s, isos. Δ)

$\angle OAP = 90°$ (tangent $\perp$ radius)

$\angle BAP = 90° - \angle ABC$

$\angle BPQ + \angle PBA + \angle BAP = 180°$ ($\angle$ sum of Δ)

$\angle BPQ = 180° - \angle PBA - \angle BAP$

$\angle BPQ = 180° - \angle ABC - \angle BAP = 90°$

$\angle OCQ = 90°$ (tangent $\perp$ radius)

$\because \angle BPQ + \angle BCQ = 90° + 90° = 180°$

$\therefore$ $PBCQ$ is a cyclic quadrilateral. (opp. $\angle$s supp.)