CIRCLE

Form 5
Vol 4

Part 4 – Cyclic quadrilateral

1. C 2. B 3. C 4. D 5. D

1. C

$\angle BDE + \angle BAE = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle BDE + 92^\circ = 180^\circ$

$\angle BDE = 88^\circ$

$\angle BED + \angle BCD = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle BED + 108^\circ = 180^\circ$

$\angle BDE = 72^\circ$

$\angle DBE + \angle BDE + \angle BDE = 180^\circ$ ($\angle$ sum of Δ)

$\angle DBE + 88^\circ + 72^\circ = 180^\circ$

$\angle DBE = 20^\circ$

2. B

$\angle SPQ = 100^\circ$ (ext. $\angle$, cyclic quad.)

$\angle PSQ = \angle PQS$ (base $\angle$s, isos. Δ)

$\angle PSQ + \angle PQS + \angle SPQ = 180^\circ$ ($\angle$ sum of Δ)

$2\angle PSQ + 100^\circ = 180^\circ$

$\angle PSQ = 40^\circ$

3. C

$\angle ABD = 90^\circ$ ($\angle$ in semi-circle)

$\angle DBC + \angle ABD + \angle ABE = 180^\circ$ (adj. $\angle$s on st. line)

$\angle DBC + 90^\circ + 70^\circ = 180^\circ$

$\angle DBC = 20^\circ$

$\angle ADB = \angle DBC = 20^\circ$ (alt. $\angle$s, $AD // BC$)

$\angle ADC = 70^\circ$ (ext. $\angle$, cyclic quad.)

$\angle BDC = \angle ADC - \angle ADB = 70^\circ - 20^\circ = 50^\circ$

4. D

$\angle EAB = \angle CDB = 3\theta + 10^\circ$ ($\angle$s in the same segment)

$\angle EAB + \angle EBA + \angle AEB = 180^\circ$ ($\angle$ sum of Δ)

$3\theta + 10^\circ + \theta + 24^\circ + 2\theta - 16^\circ = 180^\circ$

$6\theta = 162^\circ$

$\theta = 27^\circ$

5. D

$\angle ADC = \angle AFB + \angle DCF$ (ext. $\angle$ of Δ)

$\angle ADC = 30^\circ + 45^\circ = 75^\circ$

$x + \angle ADC = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$x + 75^\circ = 180^\circ$

$x = 105^\circ$

$\angle BCE = \angle DCF = 45^\circ$ (vert. opp. $\angle$s)

$y + \angle BCE = x$ (ext. $\angle$ of Δ)

$y + 45^\circ = 105^\circ$

$y = 60^\circ$

$\therefore x + y = 165^\circ$

6. $\angle OAB = \angle OBA$ (base $\angle$s, isos. Δ)

$66^\circ + 2\angle OBA = 180^\circ$ ($\angle$ sum of Δ)

$\angle OBA = 57^\circ$

$\angle ABC + 75^\circ = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle ABC = 105^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle OBC = \angle ABC - \angle OBA = 48^\circ$

7. $\angle DAC = 90^\circ$ ($\angle$ in semi-circle)

$\angle BCD + \angle DAB = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle BCD = 78^\circ$

$\angle CBP + 32^\circ = 78^\circ$ (ext. $\angle$ of Δ)

$\angle CBP = 46^\circ$

8. $\angle RPQ = 22^\circ$ ($\angle$s in the same segment)

$\angle PQR = \angle PRQ$ (base $\angle$s, isos. Δ)

$2\angle PQR + 22^\circ = 180^\circ$ ($\angle$ sum of Δ)

$\angle PQR = 79^\circ$

$\angle RST = 79^\circ$ (ext. $\angle$, cyclic quad.)

9. (a) Join CD,

$\angle DCB + 114° = 180°$ (opp. $\angle$s, cyclic quad.)

$\angle DCB = 66°$

$\angle DEF = 66°$ (ext. $\angle$, cyclic quad.)

(b) $\angle CDE + 80° = 180°$ (opp. $\angle$s, cyclic quad.)

$\angle CDE = 100°$

$\angle ABC = 100°$ (ext. $\angle$, cyclic quad.)

10. (a) $\angle CPD = \angle CQD$ ($\angle$s in the same segment)

$\angle AQB = \angle CQD$ (vert. opp. $\angle$s)

$\angle AQB = \angle APB = 25°$ ($\angle$s in the same segment)

$\angle CPD = 25°$

(b) $\angle QPD = 180° - 118° = 62°$ (opp. $\angle$s, cyclic quad.)

$\angle QPC = 62° - 25° = 37°$

$\angle BPQ = \angle BAQ$ ($\angle$s in the same segment)

$= 180° - 126° - 25°$ ($\angle$ sum of Δ)

$= 29°$

$\angle BPC = 37° + 29° = 66°$

11. (a) Since I is the incentre of ΔPSU, we have

$\angle RPS = \angle QPR$

$= \angle QSR$ ($\angle$s in the same segment)

$= \angle PSQ$

$\therefore \angle QPS = \angle RSP$

$= \dfrac{180° - 44°}{2}$ ($\angle$ sum of Δ)

$= 68°$

$\therefore \angle RPS = \dfrac{68°}{2} = 34°$

$\angle PRU = 68° + 34°$ (ext. $\angle$ of Δ)

$= 102°$

$\angle PTS = 102°$ (ext. $\angle$s, cyclic quad.)

(b) $\angle PRS = 180° - 102°$ (opp. $\angle$s, cyclic quad.)

$= 78°$

$\angle SPT = 68° - 49°$ (ext. $\angle$ of Δ)

$= 19°$

$\angle SRT = 19°$ ($\angle$s in the same segment)

$\angle PRT = 78° - 19° = 59°$

12. (a) $\angle QPT = 180^\circ - \angle QRT$ (opp.$\angle$s, cyclic quad.)

Join QS.

$\angle SQP = \angle STV$ (ext. $\angle$s, cyclic quad.)

$= \angle QRT$

$\because$ $\angle SQP + \angle QPT = \angle QRT + 180^\circ - \angle QRT = 180^\circ$

$\therefore$ $PV // QS$ (int. $\angle$s supp.)

$\angle PSR = 54^\circ$ ($\angle$s in the same segment)

$\angle QSR = \angle PSQ$ (equal chords, equal $\angle$s)

$\angle QSR = \dfrac{54^\circ}{2} = 27^\circ$

$\angle SVT = 27^\circ$ (corr. $\angle$s, $PT // QS$)

(b) Cancel

Part 5 - Tangent

1.	B	2.	A	3.	C	4.	C	5.	B
6.	A	7.	B	8.	B	9.	A	10.	A
11.	A	12.	C	13.	A	14.	B	15.	D

1. B

$\angle DCG = \angle FDG$ and $\angle CDG = \angle ECG = 55^\circ$ ($\angle$ in alt. segment)

$\angle FDC + \angle DCG + \angle DFG = 180^\circ$ ($\angle$ sum of Δ)

$(\angle FDG + 55^\circ) + \angle FDG + 65^\circ = 180^\circ$

$2\angle FDG = 60^\circ$

$\angle FDG = 30^\circ$

2. A

Join CA.

$\angle BCA = 90^\circ$ ($\angle$ in semi-circle)

$\angle DCA = \angle BCD - \angle BCA = 118^\circ - 90^\circ = 28^\circ$

$\angle ADF = \angle DCA$ ($\angle$ in alt. segment)

$\angle ADF = 28^\circ$

3. C

$\angle OQP = \angle OPQ$ (base $\angle$s, isos. Δ)

$\angle OQP + \angle OPQ + \angle POQ = 180^\circ$

$2\angle OQP + \angle OPQ + 110^\circ = 180^\circ$

$2\angle OQP = 70^\circ$

$\angle OQP = 35^\circ$

$\angle BPQ = \angle OQP$ ($\angle$ in alt. segment)

$\angle BPQ = 35^\circ$

4. C

$\angle ACB = \angle QAB$ ($\angle$ in alt. segment)

$\angle ACB + \angle QAC + \angle AQB = 180^\circ$ ($\angle$ sum of Δ)

$3\angle ACB + 36^\circ = 180^\circ$

$3\angle ACB = 144^\circ$

$\angle ACB = 48^\circ$

$\angle ACP = \angle QCP - \angle ACB = 90^\circ - 48^\circ = 42^\circ$

5. B

$\angle SBA + \angle TAB = 180^\circ$ (int. $\angle$s, $TP // SQ$)

$\angle SBA + 108^\circ = 180^\circ$

$\angle SBA = 72^\circ$

$\angle ACB = \angle SBA = 72^\circ$ ($\angle$ in alt. segment)

$\angle ABC = \angle ACB = 72^\circ$ (base $\angle$s, isos. Δ)

$\angle CBQ + \angle SBA + \angle ABC = 180^\circ$ (adj. $\angle$s on st. line)

$\angle CBQ + 72^\circ + 72^\circ = 180^\circ$

$\angle CBQ = 36^\circ$

6. A

$\angle BCA = \angle BAQ = 65^\circ$ ($\angle$ in alt. segment)

$\angle BAC = \angle BCA = 65^\circ$ (base $\angle$s, isos. Δ)

$\angle ABC + \angle BAC + \angle BCA = 180^\circ$ ($\angle$ sum of Δ)

$\angle ABC + 65^\circ + 65^\circ = 180^\circ$

$\angle ABC = 50^\circ$

7. B

$\angle AEB = \angle DBE = 90^\circ$ ($\angle$ in semi-circle)

$\angle EBC + \angle ECB = \angle AEB$ (ext. $\angle$ of Δ)

$\angle EBC + 58^\circ = 90^\circ$

$\angle EBC = 32^\circ$

$\angle BDE = \angle EBC = 32^\circ$ ($\angle$ in alt. segment)

$\angle BED + \angle BDE + \angle DBE = 180^\circ$ ($\angle$ sum of Δ)

$\angle BED + 32^\circ + 90^\circ = 180^\circ$

$\angle BED = 58^\circ$

8. B

$OB^2 + OA^2 = AB^2$

$OB^2 + 8^2 = 12^2$

$OB = \sqrt{80}$ cm

Let r cm be the radius of the circle.

$12r = 8\sqrt{80}$

$r = \frac{2}{3}\sqrt{80} \approx 5.96284794$

$r \approx 5.96$

9. A

$\angle CBD = \angle BAC = 46^\circ$ ($\angle$ in alt. segment)

$\angle CDB = \angle CBD = 46^\circ$ (base $\angle$s, isos. Δ)

$\angle ACB = \angle CDB + \angle CBD$ (ext. $\angle$ of Δ)

$\angle ACB = 46^\circ + 46^\circ = 92^\circ$

10. A

$\angle ACB = \angle BCT = 90^\circ$ ($\angle$ in semi-circle)

$\angle BAT = \angle CBT$ ($\angle$ in alt. segment)

$\Delta ABC \sim \Delta BTC$ (AA)

$\frac{BC}{AC} = \frac{TC}{BC}$ (corr. sides, ~Δs)

$\frac{BC}{12} = \frac{9}{BC}$

$BC^2 = 108$

$BC = \sqrt{108} = 6\sqrt{3}$ cm

11. A

I is true.

$CO = OA$ and $OD // AB$

$\therefore BD = DC$ (intercept thm.)

II is true.

$\angle APB = \angle BPC$ (common)

$\angle ABP = \angle BCP$ ($\angle$ in alt. segment)

$\Delta APB \sim \Delta BPC$ (AA)

III is not true.

$\angle ABC = 90^\circ$ ($\angle$ in semi-circle)

$\angle ODB + \angle ABC = 180^\circ$ (int. $\angle$s, $OQ // AB$)

$\angle ODB + 90^\circ = 180^\circ$

$\angle ODB = 90^\circ$

$\because \angle BAC \neq 90^\circ$

$\angle BAC + \angle ODB \neq 180^\circ$

$\therefore A, O, D$ and B are not concyclic.

12. C

Join CA.

$\angle BAC = \angle CAD$ (equal chords, equal $\angle$s)

$\angle BAP = 90^\circ$ (tangent $\perp$ radius)

$\angle BAC + \angle CAD + \angle DAP = 90^\circ$

$2\angle CAD + 42^\circ = 90^\circ$

$2\angle CAD = 48^\circ$

$\angle CAD = 24^\circ$

$\angle CBA = \angle CAP$ ($\angle$ in alt. segment)

$\angle CBA = \angle CAD + \angle DAP = 24^\circ + 42^\circ = 66^\circ$

$\angle ADC + \angle CBA = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle ADC + 66^\circ = 180^\circ$

$\angle ADC = 114^\circ$

13. A

I is true.

$\angle ATC = \angle BTA$ (common)

$\angle TAC = \angle TBA$ ($\angle$ in alt. segment)

$\Delta TAC \sim \Delta TBA$ (AA)

II is not true.

If $\angle BAC = 90^\circ$, then ΔACT is an obtuse-angled triangle.

III is not true.

If $\angle BAC = 90^\circ$, then $\angle ACT$ is obtuse.

14. B

Join BC.

$\angle ABE = \angle DCE = 37^\circ$ (equal arcs, equal $\angle$s)

$\angle ECB = \angle EBT$ ($\angle$ in alt. segment)

$\angle ECB = \angle ABE + \angle ABT = 37^\circ + 39^\circ = 76^\circ$

$\angle EBC + \angle ECB + \angle BEC = 180^\circ$ ($\angle$ sum of Δ)

$\angle EBC + 76^\circ + 44^\circ = 180^\circ$

$\angle EBC = 60^\circ$

$\angle ECS = \angle EBC = 60^\circ$ ($\angle$ in alt. segment)

$\angle DCS + \angle DCE = 60^\circ$

$\angle DCS + 37^\circ = 60^\circ$

$\angle DCS = 23^\circ$

15. D

Join *SR*.

$\angle TPI = \angle SPI$ and $\angle PTI = \angle STI$ (incentre)

$\angle TPI + \angle PTI + \angle PIT = 180^\circ$ ($\angle$ sum of Δ)

$\angle TPI + \angle PTI + 112^\circ = 180^\circ$

$\angle TPI + \angle PTI = 68^\circ$

$\angle TPS + \angle PTS + \angle PST = 180^\circ$ ($\angle$ sum of Δ)

$2\angle TPI + 2\angle PTI + \angle PST = 180^\circ$

$2(68^\circ) + \angle PST = 180^\circ$

$\angle PST = 44^\circ$

$\angle PTS = \angle PST = 44^\circ$ (base $\angle$s, isos. Δ)

$\therefore \angle TPS = 92^\circ$

$\angle QRS = \angle TPS = 92^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle PRS = \angle PST = 44^\circ$ ($\angle$ in alt. segment)

$\angle PRQ = \angle QRS - \angle PRS = 92^\circ - 44^\circ = 48^\circ$