CIRCLE

Form 5
Vol 4

Part 3 – Relations between arc, chord, angles

1.	A	2.	D	3.	D	4.	B	5.	C
6.	C	7.	C	8.	A	9.	B	10.	A
11.	C	12.	C	13.	B	14.	B	15.	A

1. A

$\angle COD = \angle BOC = \angle AOB = x$ (equal arcs, equal $\angle$s)

$\angle COD + \angle BOC + \angle AOB = 180^\circ$ (adj. $\angle$s on st. line)

$3x = 180^\circ$

$x = 60^\circ$

2. D

$AB = BC = CD = DE$

$\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE}$ (equal chords, equal arcs)

$\therefore \overset{\frown}{AC} = \overset{\frown}{CE} = 2\overset{\frown}{BC}$

$\angle CEA = \angle CAE = 2\angle BAC$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle CEA = \angle CAE = 2x$

$\angle CEA + \angle CAE + \angle ACE = 180^\circ$ ($\angle$ sum of Δ)

$4x + y = 180^\circ$

3. D

Join AD and OC.

$\angle COD : \angle AOD = 5 : 9$ (arcs prop. to $\angle$s at centre)

$\angle COD = 90^\circ \times \frac{5}{9} = 50^\circ$

$\angle CAD = \frac{1}{2}\angle COD = \frac{1}{2} \times 50^\circ = 25^\circ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle ADB : \angle CAD = 1 : 5$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle ADB = \frac{1}{5}\angle CAD = \frac{1}{5} \times 25^\circ = 5^\circ$

$\angle APD + \angle ADB + \angle CAD = 180^\circ$ ($\angle$ sum of Δ)

$\angle APD + 5^\circ + 25^\circ = 180^\circ$

$\angle APD = 150^\circ$

4. B

$\angle EBC : \angle ECB = 3:2$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle EBC + \angle ECB + \angle BEC = 180^\circ$ ($\angle$ sum of Δ)

$\angle EBC + \frac{2}{3}\angle EBC + 130^\circ = 180^\circ$

$\frac{5}{3}\angle EBC = 50^\circ$

$\angle EBC = 30^\circ$

5. C

Let D be a point on the circle such that AD is diameter of the circle.

$\angle ABD = 90^\circ$ ($\angle$ in semi-circle)

$\overset{\frown}{AD} : \overset{\frown}{AC} = 2:1$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\overset{\frown}{AD} = 32$ cm

Thus, the circumference of teh circle is $32 \times 2 = 64$ cm.

6. C

Join OR.

$\angle POR = 2\angle PSR = 136^\circ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

reflex$\angle POR = 224^\circ$ ($\angle$s at a pt.)

$\angle ROQ$: reflex$\angle POR = 1:7$ (arcs prop. to $\angle$s at centre)

$\angle ROQ = \frac{1}{7}\text{reflex}\angle POR = \frac{1}{7} \times 224^\circ = 32^\circ$

$\angle POQ + \angle ROQ + \text{reflex}\angle POR = 360^\circ$ ($\angle$s at a pt.)

$\angle POQ + 32^\circ + 224^\circ = 360^\circ$

$\angle POQ = 104^\circ$

7. C

$\angle COB + \angle AOB = 180^\circ$ (adj. $\angle$s on st. line)

$\angle COB + 140^\circ = 180^\circ$

$\angle COB = 40^\circ$

$\angle DOA : \angle COB = 5:4$ (arcs prop. to $\angle$s at centre)

$\angle DOA = \frac{5}{4}\angle COB = \frac{5}{4} \times 40^\circ = 50^\circ$

8. A

$\angle CAB = \angle CDB = 90^\circ$ ($\angle$ in semi-circle)

$\angle ABC + \angle CAB + \angle ACB = 180^\circ$ ($\angle$ sum of Δ)

$\angle ABC + 90^\circ + 15^\circ = 180^\circ$

$\angle ABC = 75^\circ$

$\angle CBD + \angle CDB + \angle BCD = 180^\circ$ ($\angle$ sum of Δ)

$\angle CBD + 90^\circ + 60^\circ = 180^\circ$

$\angle CBD = 30^\circ$

$\overset{\frown}{AC} : \overset{\frown}{CD} = \angle ABC : \angle CBD$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\overset{\frown}{AC} : \overset{\frown}{CD} = 5 : 2$

9. B

I is not true unless AOC and BOD are straight lines.

II is true

$\overset{\frown}{AB} : \overset{\frown}{CD} = x : y = 2 : 3$ (arcs prop. to $\angle$s at centre)

$\therefore 3\overset{\frown}{AB} = 2\overset{\frown}{CD}$

III is not true.

10. A

Join OE.

$\angle OED = \angle ODE = 65^\circ$ (base $\angle$s, isos. Δ)

$\angle DOE + \angle OED + \angle ODE = 180^\circ$ ($\angle$ sum of Δ)

$\angle DOE + 65^\circ + 65^\circ = 180^\circ$

$\angle DOE = 50^\circ$

$\angle AOE = \angle DOE = 50^\circ$ (equal arcs, equal $\angle$s)

reflex$\angle AOD = 260^\circ$ ($\angle$s at a pt.)

$\angle BOC : \text{reflex}\angle AOD = 2 : 10 = 1 : 5$ (arcs prop. to $\angle$s at centre)

$\angle BOC = \frac{1}{5}\text{reflex}\angle AOD = \frac{1}{5} \times 260^\circ = 52^\circ$

11. C

Join AB.

Let $\overset{\frown}{BED} = 3k$ and $\overset{\frown}{BCD} = 2k$ respectively.

Then $\overset{\frown}{AD} = 2.5k$ and $\overset{\frown}{AB} = 0.5k$

$\because \overset{\frown}{BC} : \overset{\frown}{AB} = 2:1$

$\therefore \overset{\frown}{BC} = k$ and thus $\overset{\frown}{CD} = k$

$\angle ABD = 90^\circ$ ($\angle$ in semi-circle)

$\angle CBD : \angle ABD = 1:2.5$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle CBD = \frac{1}{2.5}\angle ABD = \frac{1}{2.5} \times 90^\circ = 36^\circ$

12. C

Join CE.

$\angle ACE = 90^\circ$ ($\angle$ in semi-circle)

$\because CD = DE$

$\therefore \overset{\frown}{CD} = \overset{\frown}{DE}$ (equal chords, equal arcs)

$\therefore \overset{\frown}{AE} : \overset{\frown}{DE} = 5:1$

$\angle ECD : \angle ACE = 1:5$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle ECD = \frac{1}{5}\angle ACE = \frac{1}{5} \times 90^\circ = 18^\circ$

$\angle ACD = \angle ACE + \angle ECD = 90^\circ + 18^\circ = 108^\circ$

13. B

$\angle CBA : \angle BAD = 13:8$ (arcs prop. to $\angle$s at $\odot^{ce}$)

Let $\angle CBA = 13k$ and $\angle BAD = 8k$

$\angle BDA = 90^\circ$ ($\angle$ in semi-circle)

$\angle DBA + \angle BAD + \angle BDA = 180^\circ$ ($\angle$ sum of Δ)

$(13k - 36^\circ) + 8k + 90^\circ = 180^\circ$

$k = 6$

$\therefore \angle BAD = 8k = 48^\circ$

$\angle DAC = \angle CBD = 36^\circ$ ($\angle$s in the same segment)

$\therefore \angle CAB = \angle BAD - \angle DAC = 48^\circ - 36^\circ = 12^\circ$

$\angle CDB = \angle CAB = 12^\circ$ ($\angle$s in the same segment)

$\angle BEC + \angle EDA + \angle BAD = 180^\circ$ ($\angle$ sum of Δ)

$\angle BEC + (12^\circ + 90^\circ) + 48^\circ = 180^\circ$

$\angle BEC = 30^\circ$

14. B

Join AB.

reflex$\angle AOD = 270^\circ$ ($\angle$s at a pt.)

$\angle ABD = \frac{1}{2}\text{reflex}\angle AOD = 135^\circ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle CAB + \angle ABD = \angle AED$ (ext. $\angle$ of Δ)

$\angle CAB + 135^\circ = 159^\circ$

$\angle CAB = 24^\circ$

$\angle BCE : \angle CAB = 1:2$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle BCE = \frac{1}{2}\angle CAB = \frac{1}{2} \times 24^\circ = 12^\circ$

15. A

I is not true.

II is not true.

III is true.

$\angle ABC = 90^\circ$ ($\angle$ in semi-circle)

$\because \overset{\frown}{AB} : \overset{\frown}{BC} : \overset{\frown}{CD} = 7:8:10$

$\therefore \overset{\frown}{AD} : \overset{\frown}{AC} = 5:15 = 1:3$

$\angle ACD : \angle ABC = 1:3$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle ACD = \frac{1}{3}\angle ABC = \frac{1}{3} \times 90^\circ = 30^\circ$