CIRCLE

Form 5
Vol 4

Part 1 – Chords in circle

1.	D	2.	C	3.	B	4.	A	5.	B
6.	B	7.	C						

1. D

$\because CN = ND$, $\therefore AB \perp CD$ (line joining centre to mid-pt. of chord $\perp$ chord)

$ON^2 + CN^2 = OC^2$

$ON = \sqrt{5^2 - 4^2} = 3$ cm

$AN = OA + ON = 5 + 3 = 8$ cm

2. C

$\because OP \perp AB$, $\therefore AM = MB$ (line from centre $\perp$ chord bisects chord)

Let r cm be the radius of the circle.

$OM^2 + AM^2 = OA^2$

$(r-4)^2 + 8^2 = r^2$

$-8r + 16 + 64 = 0$

$r = 10$

3. B

$\because ON \perp PQ$, $\therefore PN = NQ$ (line from centre $\perp$ chord bisects chord)

$ON^2 + PN^2 = OP^2$

$ON = \sqrt{13^2 - 5^2} = 12$ cm

4. A

$\because OM \perp AB$, $\therefore AM = MB$ (line from centre $\perp$ chord bisects chord)

$OM^2 + AM^2 = OA^2$

$OM = \sqrt{6^2 - 5^2} = \sqrt{11}$ cm

5. B

$\because OM \perp AB$, $ON \perp CD$ and $AB = CD$, $\therefore OM = ON$ (equal chords, equidistance from centre)

Note that $OMEN$ is a square.

$OE = \sqrt{2OM^2} = \sqrt{22}$ cm

6. B

$\because OQ \perp CD, \therefore CQ = QD$ (line from centre $\perp$ chord bisects chord)

$OQ^2 + CQ^2 = OC^2$

$OC = \sqrt{8^2 + 6^2} = 10$ cm

$\because AP = PB, \therefore OP \perp AB$ (line joining centre to mid-pt. of chord $\perp$ chord)

$OP^2 + AP^2 = OA^2$

$AP = \sqrt{10^2 - 6^2} = 8$ cm

$AB = 2AP = 16$ cm

7. C

Denote the centre by O, midpoint of AC by M and midpoint of BC by N respectively.

$\therefore OM \perp AC$ and $ON \perp BC$ (line joining centre to mid-pt. of chord $\perp$ chord)

Note that $OMCN$ is a rectangle.

$OM^2 + MC^2 = OC^2$

$OC = \sqrt{6^2 + 8^2} = 10$ cm

$\therefore$ radius = 10 cm

Alternatively,

$\because \angle BCA = 90^\circ, \therefore AB$ is a diameter. (converse of $\angle$ in semi-circle)

$AC^2 + BC^2 = AB^2$

$AB = \sqrt{16^2 + 12^2} = 20$ cm

$\therefore$ radius = 10 cm

Part 2 – Angles in circle

1.	B	2.	A	3.	D	4.	B	5.	B
6.	B	7.	B	8.	B				

1. B

$\angle AOB = \angle APB = x$ (opp. $\angle$s of // gram)

reflex$\angle AOB = 2\angle APB = 2x$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\therefore x + 2x = 360^\circ$ ($\angle$s at a pt.)

$x = 120^\circ$

2. A

reflex$\angle AOC = 360^\circ - 118^\circ = 242^\circ$ ($\angle$s at a pt.)

$\angle ABC = \dfrac{\text{reflex}\angle AOC}{2} = 121^\circ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle BAC + \angle ABC + \angle BCA = 180^\circ$ ($\angle$ sum of Δ)

$\angle BAC + 121^\circ + 36^\circ = 180^\circ$

$\angle BAC = 23^\circ$

3. D

$\angle ABC = \angle ACB = 60^\circ$ (prop. of equil. Δ)

$\angle AOC = 2\angle ABC = 120^\circ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$OA = OC$ (radii)

$\angle OCA = \angle OAC$ (base $\angle$s, isos. Δ)

$\angle OCA + \angle OAC + \angle AOC = 180^\circ$ ($\angle$ sum of Δ)

$2\angle OCA + 120^\circ = 180^\circ$

$\angle OCA = 30^\circ$

$\angle BCO = \angle ACB - \angle OCA = 60^\circ - 30^\circ = 30^\circ$

Alternatively,

Join OB.

Note that $\Delta OCB \cong \Delta OCA \cong \Delta OAB$ (SSS)

$\therefore \angle BCO = \angle ACO$ (corr. $\angle$s, $\cong \Delta$s)

$\angle ACB = 60^\circ$ (prop. of equil. Δ)

$2\angle BCO = 60^\circ$

$\angle BCO = 30^\circ$

4. B

$OB = OA$ (radii)

$\therefore \Delta OAB$ is equil. Δ

$\angle AOB = 60^\circ$ (prop. of equil. Δ)

$\angle ACB = \frac{1}{2}\angle AOB$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle ACB = 30^\circ$

5. B

Join BC. Let $\angle ABD = x$.

$\because AD = DB$

$\because \angle DAB = \angle DBA = x$ (base $\angle$s, isos. Δ)

$\angle DAC = x - 16^\circ$

$\angle DBC = \angle DAC = x - 16^\circ$ ($\angle$s in the same segment)

$\angle ABC = 90^\circ$ ($\angle$ in semi-circle)

$x + x - 16^\circ = 90^\circ$

$x = 53^\circ$

6. B

$\angle CAD = 90^\circ$ ($\angle$ in semi-circle)

$\angle BAC = \angle BAD - \angle CAD = 26^\circ$

$\angle EDC = \angle BAC = 26^\circ$ ($\angle$s in the same segment)

$\angle ACD + \angle EDC = \angle CFE$ (ext. $\angle$ of Δ)

$\angle ABD + \angle BAC + 26^\circ = 78^\circ$ (ext. $\angle$ of Δ)

$\angle ABD + 26^\circ + 26^\circ = 78^\circ$

$\angle ABD = 26^\circ$

7. B

$\angle ABE + \angle OAB = \angle BFC$ (ext. $\angle$ of Δ)

$\angle ABE + 28^\circ = 74^\circ$

$\angle ABE = 46^\circ$

$\angle AOE = 2\angle ABE = 92^\circ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle CDE = \frac{1}{2}\text{reflex}\angle AOE$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle CDE = \frac{1}{2}(\angle AOE + \angle AOC) = \frac{1}{2}(92^\circ + 180^\circ) = 136^\circ$

8. B

Join CA and CE.

$\angle AEB + \angle DAE = \angle AFB$ (ext. $\angle$ of Δ)

$\angle AEB + 18^\circ = 92^\circ$

$\angle AEB = 74^\circ$

$\angle ACB = \angle AEB = 74^\circ$ ($\angle$s in the same segment)

$\angle ACE = \angle ABE = \angle ADE = 27^\circ$ ($\angle$s in the same segment)

$\angle ECD = \angle EAD = 18^\circ$ ($\angle$s in the same segment)

$\angle ABC + \angle BCD = 180^\circ$ (int. $\angle$s, $AB // DC$)

$(\angle CBE + 27^\circ) + (74^\circ + 27^\circ + 18^\circ) = 180^\circ$

$\angle CBE = 34^\circ$