超研數學 ASPIRE MATHEMATICS

SOLUTION

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SUMMER QUIZ 02

Form 6
AS & GS

Structural Questions

1. (a) $\frac{\left[2(-63)+(n-1)(14)\right]n}{2}=525$ 1M

$14n^2-140n-1050=0$

$n=15$ or $n=-5$ (rej.) 1M

$\therefore$ Number of terms = 15 1A

(b) $\frac{(-63+m)(15)}{2}=525$ 1M

$m=133$ 1A

(5)

2. $r=-\frac{5}{3}$ 1M

$-405\left(-\frac{5}{3}\right)^{m-1}>80000000$ 1M

$\left(-\frac{5}{3}\right)^{m-1}<-\frac{80000000}{405}$

$-\left(\frac{5}{3}\right)^{m-1}<-\frac{80000000}{405}$ ($\because m-1$ is odd)

$\left(\frac{5}{3}\right)^{m-1}>\frac{80000000}{405}$ 1M

$(m-1)\log\left(\frac{5}{3}\right)>\log\left(\frac{80000000}{405}\right)$ 1M

$m-1>23.9$

$m>24.9$

$\therefore m=26$ ($\because m$ is even) 1A

(5)

3. $T(n)=3n-53$

$S(n)<180$

$\frac{(-50+3n-53)n}{2}<180$ 1M

$3n^2-103n-360<0$

$-3.20<n<37.5$ 1M

$\because n\geq 1$

$\therefore 1\leq n<37.5$ 1M can be absorbed

$\therefore$ There are 37. 1A

(4)

4. (a) $\frac{ar}{1-r^2}=3$ and $\frac{ar^2}{1-r^3}=\frac{12}{13}$ 1M

$\frac{(1-r^2)r}{1-r^3}=\frac{4}{13}$ 1M

$\frac{(1+r)r}{1+r+r^2}=\frac{4}{13}$ 1M

$13r+13r^2=4+4r+4r^2$

$9r^2+9r-4=0$

$r=\frac{1}{3}$ or $r=-\frac{4}{3}$ (rej.)

$\therefore$ The common ratio $=\frac{1}{3}$ 1A

(b) For $r=\frac{1}{3}$,

$\frac{a\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}=3$ 1M

$\therefore a=8$

$\therefore T(1)+T(2)+T(3)+T(4)+T(5)+\cdots$

$=\frac{8}{1-\frac{1}{3}}$

$=12$ 1A

(c) $A(n) = \log[T(n)]$

$A(n+1) - A(n)$ 1M

$= \log[T(n+1)] - \log[T(n)]$

$= \log\left[8\left(\frac{1}{3}\right)^n\right] - \log\left[8\left(\frac{1}{3}\right)^{n-1}\right]$

$= \log\frac{1}{3} = -\log 3$ 1A

$\therefore A(n)$ is an arithmetic sequence. 1A f.t.

(9)

5. (a)(i) The amount he owes $= \$\left(4\times10^5\times(1+r\%) - 6\times10^4\right)$ 1A

(ii) $\left[4\times10^5\times(1+r\%) - 6\times10^4\right]\times(1+r\%) - 6\times10^4 = 3.58\times10^5$ 1M

$4\times10^5(1+r\%)^2 - 6\times10^4(1+r\%) - 4.18\times10^5 = 0$ 1M

$(1+r\%) = 1.1$ or $(1+r\%) = -0.95$ (rej.)

$r = 10$ 1A

(4)

(b)(i) The amount he owes

$= 4\times10^5(1+r\%)^{n-1} - 6\times10^4\frac{\left[(1+r\%)^{n-1} - 1\right]}{\left[(1+r\%) - 1\right]}$ 1M

$= \$\left(6\times10^5 - 2\times10^5(1.1)^{n-1}\right)$ 1A

(ii) $6\times10^5 - 2\times10^5(1.1)^{n-1} < 2\times10^5$

$2\times10^5(1.1)^{n-1} > 4\times10^5$

$1.1^{n-1} > 2$

$(n-1)\log 1.1 > \log 2$ 1M

$n > 8.272540897\ldots$

$\therefore n = 9$

$\therefore$ At the start of 9th year. 1A

(4)

(c) $\begin{cases} a+b(1.21)=126400 \\ a+b(1.21)^2=85744 \end{cases}$

Solving, we have

$a=320000$ and $b=-160000$ 1M

$320000-160000(1.21)^{m-1}<6\times10^5-2\times10^5(1.1)^{m+7}$

$\frac{4}{1.1^2}\left((1.1)^m\right)^2-5\times1.1^7(1.1)^m+7>0$ 1M

$1.1^m<1.240602098$ or $1.1^m>1.706832516$

$m\log1.1<\log1.240602098$ or $m\log1.1>\log1.706832516$

$m<2.262054538$ or $m>5.609467152$ 1M

The amount of he owes for the 2nd car is NOT less than for the 1st car in the 3rd year.

So, incorrect. 1A f.t.

(4)