4 CENTRE OF TRIANGLE

Form 5
Vol 2

Part E – 4 Centre: Incentre

1. Let $\angle IBC = \theta, \angle ICB = \phi$

I is incenter of $\Delta, \angle IBC = \angle IBA = \theta, \angle ICB = \angle ICA = \phi$

$\therefore \theta + \theta + \phi + \phi + 68° = 180°,$

$\theta + \phi = 56°$

From ΔIBC,

$\angle BIC = 180° - (\phi + \theta)$

$= 180° - 56°$

$= 124°$

2. $AC = \sqrt{8^2 + 6^2} = 10$

$\because 8 - R + 6 - R = 10$

$R = 2$

Alternative solution

Since $pr = 2a$, $p = 24$ and $a = 24$

$\therefore r = 2$

3. $AC = \sqrt{(9-4)^2 + (18-6)^2} = 13$

$BC = \sqrt{(14-9)^2 + (18-6)^2} = 13$

$AB = 10$

Area of $\Delta ABC = \dfrac{10 \times 12}{2} = 60$

$pr = 2a$

$36r = 2(60)$

$r = \dfrac{10}{3}$

Radius $= \dfrac{10}{3}$

4. Note that A lies on the line $x+5y=a$

$$\tan\frac{\angle BAO}{2}=\frac{1}{5}$$

$$\tan\angle BAO=\frac{5}{12}$$

$$\frac{b}{a}=\frac{5}{12}$$

$\therefore a:b=12:5$

5. Let the coordinate of the incenter of ΔOAB be $I:(R,R)$

Draw three perpendicular distance from I to AO, BO, AB and form point C, D, E respectively.

$\therefore CO=DO=R$ units

$AC=AE=(5-R)$ units

$BD=BE=(12-R)$ units

$\because AE+BE=AB=\sqrt{5^2+12^2}=13$ units

$\therefore 5-R+12-R=13$

$R=2$

The coordinate of the incenter of ΔOAB is $(2,2)$

Alternative solution

Since $pr=2a$, $p=13+12+5=30$ and $a=30$

$\therefore r=2$

The coordinate of the incenter of ΔOAB is $(2,2)$

6. $OA=a$, $AB=3$ and $OB=\sqrt{9+a^2}$

Area of $\Delta OAB=\frac{3}{2}a$

$$pr=2a$$

$$a+3+\sqrt{9+a^2}=3a$$

$$3a^2-12a=0$$

$$a=4$$

7. Let the coordinate of incenter be $I(x, y)$

Length of AB = 25 units, length of BO = 25 units and length of OC = 30 units
Draw three lines from I to AO, OB, AB and form point C, D, E respectively.
Where $IC \perp AO$, $ID \perp BO$, $IE \perp AB$
$\because \Delta AOB$ is an isosceles triangle.
$BC \perp AO$ and $AO = OC = 15$ units (prop. of isos. Δ)
$OC = OD = 15$ units, $AC = AE = 15$ units, $BD = BE = 25 - 15 = 10$ units (tangent prop.)
Thus, the x-coordinate of I is 15.
Since $IC \perp AO$,
Slope of $IC \times$ Slope of $OA = -1$

Coordinate of point C: (9.12)

$$\left(\frac{y-12}{15-9}\right) \times \frac{24-0}{18-0} = -1$$

$$y = \frac{15}{2}$$

The coordinate of I: $\left(15, \frac{15}{2}\right)$

8. Note that $(5,0)$ is the vertex of the triangle,
and the triangle is isosceles with $x = 5$ is the axis of symmetry.
Thus, the coordinates of incentre are $(5, 4.25)$
Radius $= k - 4.25$

$$\left|\frac{8(5)+15(4.25)-40}{\sqrt{8^2+15^2}}\right| = k - 4.25$$

$$k = 8$$

9. y-int of $L_1 = 2$

Thus, equation of L_2 is $12x + 5y - 10 = 0$

Note that the triangle is isosceles with $x = 0$ is the axis of symmetry.
Thus, the coordinates of incentre are $(0, 15)$
Radius $= a - 15$

$$\left|\frac{12(0)-5(15)+10}{\sqrt{12^2+5^2}}\right| = a - 15$$

$$a = 20$$