AS & GS

Form 6

Vol 1

Part 10 – Application in G.S.

1. (a) $a = 500$, $r = 0.88$

$$S(5) = \frac{500(1-0.88^5)}{1-0.88}$$
$$= 1967.8\ldots$$

$\therefore$ Total perimeter $= 1970$ cm (to 3 sig. fig.)

(b)
$$S(\infty) = \frac{500}{1-0.88}$$
$$= \frac{12500}{3}$$

$\therefore$ Total perimeter $= 4170$ cm (to 3 sig. fig.)

2.(a)(i)
$$A_1B_1 = \sqrt{4^2+4^2}$$
$$= 4\sqrt{2}$$

perimeter of $\Delta OA_1B_1 = 4+4+4\sqrt{2} = 8+4\sqrt{2}$ cm

$$OA_2 = OC_1 = A_1C_1$$
$$= \frac{A_1B_1}{2}$$
$$= 2\sqrt{2}$$

$$A_2B_2 = \sqrt{(2\sqrt{2})^2+(2\sqrt{2})^2}$$
$$= 4$$

perimeter of $\Delta OA_2B_2 = 2\sqrt{2}+2\sqrt{2}+4 = 4+4\sqrt{2}$ cm

Similarly,

$OA_3 = 2$

$$A_3B_3 = \sqrt{2^2 + 2^2}$$
$$= 2\sqrt{2}$$

perimeter of $\Delta OA_3B_3 = 2+2+2\sqrt{2} = 4+2\sqrt{2}$ cm

(ii) perimeter of sector $OA_2B_2 = 2\sqrt{2}+2\sqrt{2}+\frac{1}{4}[2\pi(2\sqrt{2})] = 4\sqrt{2}+\sqrt{2}\pi$ cm

perimeter of sector $OA_3B_3 = 2+2+\frac{1}{4}[2\pi(2)] = 4+\pi$ cm

perimeter of sector $OA_4B_4 = \sqrt{2}+\sqrt{2}+\frac{1}{4}[2\pi(\sqrt{2})] = 2\sqrt{2}+\frac{\sqrt{2}}{2}\pi$ cm

(b)(i) $a = 8+4\sqrt{2}$, $r = \frac{4+4\sqrt{2}}{8+4\sqrt{2}} = \frac{1}{\sqrt{2}}$

Total perimeter

$$= \frac{8+4\sqrt{2}}{1-\frac{1}{\sqrt{2}}}$$

$$= \frac{(8+4\sqrt{2})\left(1+\frac{1}{\sqrt{2}}\right)}{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)}$$

$$= 2(8+4\sqrt{2})\left(1+\frac{1}{\sqrt{2}}\right)$$

$$= 2(8+4\sqrt{2}+4\sqrt{2}+4)$$

$$= 24+16\sqrt{2} \text{ cm}$$

(ii) $a = 4\sqrt{2}+\sqrt{2}\pi$, $r = \dfrac{4+\pi}{4\sqrt{2}+\sqrt{2}\pi} = \dfrac{1}{\sqrt{2}}$

Perimeter of all sectors

$$= \frac{4\sqrt{2}+\sqrt{2}\pi}{1-\frac{1}{\sqrt{2}}}$$

$$= \frac{(4\sqrt{2}+\sqrt{2}\pi)\left(1+\frac{1}{\sqrt{2}}\right)}{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)}$$

$$= 2(4\sqrt{2}+\sqrt{2}\pi)\left(1+\frac{1}{\sqrt{2}}\right)$$

$$= 2(4\sqrt{2}+4+\sqrt{2}\pi+\pi)$$

$$= 8+8\sqrt{2}+(2+2\sqrt{2})\pi \text{ cm}$$

3. Raymond's total savings in successive year is a G.S.

$$\frac{(10000\times 20\%\times 12)(1.1^n-1)}{1.1-1} \le 200000$$

$$1.1^n \le \frac{11}{6}$$

$$n\log 1.1 \le \log\frac{11}{6}$$

$$n \le 6.3596\ldots$$

Raymond's total savings will not exceed in 6 years.

Raymond's monthly saving in the 7th year

$= 10000\times 20\%\times 1.1^6$

$= 3543.122$

$$\frac{24000(1.1^6-1)}{1.1-1} + 3543.122(k-72) > 200000$$

$$k > 76.184$$

$\therefore$ At least 77 months

4.(a)

$$T(1)=4000000\left(1+\frac{4\%}{12}\right)-20000$$

$$T(2)=4000000\left(1+\frac{4\%}{12}\right)^2-20000\left(1+\frac{4\%}{12}\right)-20000$$

$$T(3)=4000000\left(1+\frac{4\%}{12}\right)^3-20000\left(1+\frac{4\%}{12}\right)^2-20000\left(1+\frac{4\%}{12}\right)-20000$$

$$4000000\left(1+\frac{4\%}{12}\right)^n-20000\left(1+\frac{4\%}{12}\right)^{n-1}-20000\left(1+\frac{4\%}{12}\right)^{n-2}-\ldots-20000\le 0$$

$$4000000\left(1+\frac{4\%}{12}\right)^n-\frac{20000\left[\left(1+\frac{4\%}{12}\right)^n-1\right]}{\left(1+\frac{4\%}{12}\right)-1}\le 0$$

$$\left(1+\frac{4\%}{12}\right)^n\ge 3$$

$$n\log\left(1+\frac{4\%}{12}\right)\ge\log 3$$

$$n\ge 330.13$$

$\therefore$ 331 months

(b)

$$4000000\left(1+\frac{4\%}{12}\right)^{300}-\frac{m\left[\left(1+\frac{4\%}{12}\right)^{300}-1\right]}{\left(1+\frac{4\%}{12}\right)-1}\le 0$$

$$m\ge 21113.47$$

$\therefore$ Monthly installment $=\$21114$ (to the nearest dollar)

(c)

$$4000000\times\frac{4\%}{12}=\$13333.33$$

$$>\$12000$$

$\therefore$ The bank refuses his request.

5.(a)

$a=3000$, $r=1.03$, $n=12$

$S(12)$

$$=\frac{3000(1.03^{12}-1)}{1.03-1}$$

$$=42576$$

Total amount $=\$42600$ (to 3 sig. fig.)

(b)

$$T(1) = 3000(1.01)$$

$$T(2) = 3000(1.01)^2 + 3000(1.03)(1.01)$$

$$T(3) = 3000(1.01)^3 + 3000(1.03)(1.01)^2 + 3000(1.03)^2(1.01)$$

$$\vdots$$

$$T(12) = 3000(1.01)^{12} + 3000(1.03)(1.01)^{11} + \ldots + 3000(1.03)^{11}(1.01)$$

$$= \frac{3000(1.01)^{12}\left[\left(\frac{1.03}{1.01}\right)^{12} - 1\right]}{\frac{1.03}{1.01} - 1}$$

$$= \$45288.78$$

$$< \$48000$$

$\therefore$ disagree

6.(a)(i)

$$P(1) = ab$$

$$ab = 13800 \text{......(1)}$$

$$P(3) = ab^3$$

$$ab^3 = 19872 \text{......(1)}$$

$$\therefore \quad b^2 = 1.44$$

$$b = 1.2$$

$$a = 11500$$

Number of patients treated by X in the 5$^{\text{th}}$ year

$$P(5) = 11500(1.2)^5$$

$$= 28615.68$$

$\therefore$ Required number $= 28600$ (to 3 sig. fig.)

(ii) total number of patients

$$= \frac{11500(1.2)(1.2^n - 1)}{1.2 - 1}$$

$$= 69000(1.2^n - 1)$$

$$= 69000(1.2^n) - 69000$$

(b)(i) $Q(m+8)-P(m+13)$

$=\frac{1}{2}ab^{2(m+8)}-ab^{m+13}$

$=\frac{1}{2}ab^{2m+16}-ab^{m+13}$

$=ab^{m+13}\left(\frac{1}{2}b^{m+3}-1\right)$

For $\frac{1}{2}b^{m+3}-1$, $m\geq 1$

$\frac{1}{2}(1.2)^{m+3}-1>0$

$\therefore Q(m+8)-P(m+13)>0$

$\therefore$ agree

(ii)

$$P(n)+Q(n-5)>600000$$

$$ab^{n}+\frac{1}{2}ab^{2(n-5)}>600000$$

$$(1.2)^{2n}+2(1.2)^{10}(1.2)^{n}-\frac{2400}{23}(1.2)^{10}>0$$

$$1.2^{n}>19.96991175\ldots$$

$$n>16.423\ldots$$

$\therefore$ 17^{th} year

7.(a)(i) the required amount

$=P\left(1+\frac{3\%}{12}\right)-x$

$=\$P(1.0025)-x$

(ii) $T(1)=P(1.0025)-x$

$T(2)=P(1.0025)^{2}-1.0025x-x$

$T(3)=P(1.0025)^{3}-1.0025^{2}x-1.0025x-x$

$T(n)=P(1.0025)^{n}-\frac{x[(1.0025)^{n}-1]}{1.0025-1}$

$\quad=\$P(1.0025)^{n}-400x[(1.0025)^{n}-1]$

(b)(i) $P = 3000000 \times 70\% = 2100000$, $x = 10000$

$$2100000(1.0025)^n - 400(10000)[(1.0025)^n - 1] \le 0$$

$$19(1.0025)^n \ge 40$$

$$n \log 1.0025 \ge \log \frac{40}{19}$$

$$n \ge 298.15\ldots$$

$\therefore$ 299 months

(ii) $2100000 \times 0.0025 = \$5250$

$$> \$5000$$

$\therefore$ Not approve