

AS & GS

Form 6

Vol 1

Part 9 – Geometric Series

1. A

2. C

3. $a = -81, r = -\frac{1}{3}$

$$S(7) = \frac{-81 \left[1 - \left(-\frac{1}{3} \right)^7 \right]}{1 + \frac{1}{3}}$$

$$= -\frac{547}{9}$$

4. $S(4) = 2808$

$$\frac{a(5^4 - 1)}{5 - 1} = 2808$$

$$a = 18$$

5. $S(3) = \frac{76}{27}$

$$\frac{\frac{16}{27}(1 - r^3)}{1 - r} = \frac{76}{27}$$

$$\frac{1 - r^3}{1 - r} = \frac{19}{4}$$

$$1 + r + r^2 = \frac{19}{4}$$

$$4r^2 + 4r - 15 = 0$$

$$r = 1.5 \text{ or } -2.5$$

$$6. \quad a = 2, \quad r = \frac{8}{2} = 4$$

$$T(n) = 2048$$

$$2(4)^{n-1} = 2048$$

$$n = 6$$

sum of all the terms

$$= S(6)$$

$$= \frac{2(4^6 - 1)}{4 - 1}$$

$$= 2730$$

$$7. \quad (a) \quad S(3) = 196$$

$$\frac{28(1-r^3)}{1-r} = 196$$

$$\frac{1-r^3}{1-r} = 7$$

$$1+r+r^2 = 7$$

$$r^2 + r - 6 = 0$$

$$r = -3 \text{ or } 2 \text{ (rej.)}$$

$$(b) \quad S(k) = -5096$$

$$\frac{28[1-(-3)^k]}{1-(-3)} = -5096$$

$$1-(-3)^k = -728$$

$$(-3)^k = 729$$

$$k = 6$$

$$8. \quad a = 27, \quad r = -\frac{18}{27} = -\frac{2}{3}$$

$$S(\infty) = \frac{27}{1 - \left(-\frac{2}{3}\right)}$$

$$= \frac{81}{5}$$

$$9. \quad a = -125, \quad r = \frac{-100}{-125} = \frac{4}{5}$$

$$S(k) < -550$$

$$\frac{-125 \left[1 - \left(\frac{4}{5} \right)^k \right]}{1 - \frac{4}{5}} < -550$$

$$1 - \left(\frac{4}{5} \right)^k > \frac{22}{25}$$

$$\left(\frac{4}{5} \right)^k < \frac{3}{25}$$

$$k \log \left(\frac{4}{5} \right) < \log \left(\frac{3}{25} \right)$$

$$k > 9.5018$$

\therefore smallest value of $k = 10$

$$10. \quad S(\infty) = 10240$$

$$\frac{a}{1-r} = 10240$$

$$S(9) = 10220$$

$$\frac{a(1-r^9)}{1-r} = 10220$$

$$10240(1-r^9) = 10220$$

$$r^9 = \frac{1}{512}$$

$$r = \frac{1}{2}$$

$$\frac{a}{1-\frac{1}{2}} = 10240$$

$$a = 5120$$

$$\therefore a = 5120, \quad r = \frac{1}{2}$$