AS & GS

Form 6
Vol 1

Part 7 – Geometric Sequence

1. D

2. $\frac{T(2)}{T(1)} = \frac{-14}{7} = -2$

$\frac{T(3)}{T(2)} = \frac{28}{-14} = -2$

$\frac{T(4)}{T(3)} = \frac{-42}{28} = -\frac{3}{2}$

$\therefore$ No

3. (a) $T(n) = \frac{125}{8}\left(-\frac{2}{5}\right)^{n-1}$

$= (-1)^{n-1}\left(\frac{2}{5}\right)^{-3}\left(\frac{2}{5}\right)^{n-1}$

$= (-1)^{n-1}\left(\frac{2}{5}\right)^{n-4}$

(b) $T(5) = (-1)^{5-1}\left(\frac{2}{5}\right)^{5-4} = \frac{2}{5}$

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4. $T(3)=\frac{1}{27}$

$ar^2=\frac{1}{27}$.....(1)

$T(6)=-\frac{1}{729}$

$ar^5=\frac{1}{729}$.....(2)

$\therefore \quad r^3=-\frac{1}{27}$

$r=-\frac{1}{3}$

$a=\frac{1}{3}$

general term

$=\frac{1}{3}\left(-\frac{1}{3}\right)^{n-1}$

$=(-1)^{n-1}\left(\frac{1}{3}\right)^n$

5. $T(4)=27$

$ar^3=27$.....(1)

$T(6)=3$

$ar^5=3$.....(2)

$r^2=\frac{1}{9}$

$\therefore \quad r=\pm\frac{1}{3}$

$a=\pm 729$

case 1: $a=729$, $r=\frac{1}{3}$

general term

$=729\left(\frac{1}{3}\right)^{n-1}$

$=\left(\frac{1}{3}\right)^{n-7}$ (or 3^{7-n})

case 2: $a=-729$, $r=-\frac{1}{3}$

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general term

$= -729\left(-\frac{1}{3}\right)^{n-1}$

$= (-1)^n\left(\frac{1}{3}\right)^{n-7}$ (or $(-1)^n(3^{7-n})$)

6. $T(1)\times T(4) = 108$

$a^2r^3 = 108$.....(1)

$T(3) = 18$

$ar^2 = 18$

$a^2r^4 = 324$.....(2)

$\therefore \quad r = 3$

$a = 2$

$T(n) = 2(3)^{n-1}$

7. (a) $\frac{T(n+1)}{T(n)} = \frac{\frac{a^{n+2}b}{2}}{\frac{a^{n+1}b}{2}}$

$= a$

$\therefore \quad \frac{T(n+1)}{T(n)}$ is a constant.

(b) $\frac{T(2)}{T(1)} = \frac{9}{3}$

$a = 3$

$T(1) = 3$

$\frac{3^{1+1}b}{2} = 3$

$b = \frac{2}{3}$

$\therefore \quad a = 3, \quad b = \frac{2}{3}$

8. $T(4) + T(5) = 48$

$ar^3 + ar^4 = 48$

$ar^3(1+r) = 48$.....(1)

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$$T(2)+T(3)=3$$
$$ar+ar^2=3$$
$$ar(1+r)=3.....(2)$$

$$\therefore\ r^2=16$$
$$r=\pm 4$$

9.
$$T(2)+T(5)=72$$
$$ar+ar^4=72$$
$$ar(1+r^3)=72.....(1)$$

$$T(5)+T(8)=9$$
$$ar^4+ar^7=9$$
$$ar^4(1+r^3)=9.....(2)$$

$$\therefore\ r^3=\frac{1}{8}$$
$$r=\frac{1}{2}$$

$$a\left(\frac{1}{2}\right)^4\left[1+\left(\frac{1}{2}\right)^3\right]=9$$
$$a=128$$

$$\therefore\ 1^{st}\text{ term }=128$$

10.
$$a=2,\ r=\frac{6}{2}=3$$
$$T(n)=486$$
$$2(3)^{n-1}=486$$
$$n=6$$

11. (a)
$$a=-\frac{5}{4},\ r=\frac{5}{2}\div\left(-\frac{5}{4}\right)=-2$$

General term
$$=-\frac{5}{4}(-2)^{n-1}$$
$$=(-1)^n(5)(2)^{n-3}$$

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(b) $T(k) > 2550$

$(-1)^k (5)(2)^{k-3} > 2550$

$2^{k-3} > 510$

$(k-3)\log 2 > \log 510$

$k > 11.994$

$\therefore$ value of $k = 12$

12. (a) $a = \frac{8}{81}$, $r = \frac{4}{27} \div \frac{8}{81} = \frac{3}{2}$

General term

$T(n) = \frac{8}{81}\left(\frac{3}{2}\right)^{n-1}$

$= \frac{1}{3}\left(\frac{3}{2}\right)^{n-4}$

(b) $T(k) < \frac{81}{32}$

$\frac{1}{3}\left(\frac{3}{2}\right)^{k-4} < \frac{81}{32}$

$\left(\frac{3}{2}\right)^{k-4} < \frac{243}{32}$

$(k-4)\log\frac{3}{2} < \log\frac{243}{32}$

$k < 9$

$\therefore$ greatest value of $k = 8$

Part 8 – Properties of G.S.

1. B
2. D
3. A
4. $y^2 = 32$

   $y = \pm 4\sqrt{2}$

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5.

$$T(1)=\frac{1}{54}$$

$$a=\frac{1}{54}$$

$$T(5)=\frac{3}{2}$$

$$ar^4=\frac{3}{2}$$

$$\frac{1}{54}r^4=\frac{3}{2}$$

$$r=\pm 3$$

$$\therefore \begin{cases} x=\frac{1}{18} \\ y=\frac{1}{6} \\ z=\frac{1}{2} \end{cases}, \begin{cases} x=-\frac{1}{18} \\ y=\frac{1}{6} \\ z=-\frac{1}{2} \end{cases}$$

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